Chapter 4

Inverted Indexing for Text Retrieval

Web search is the quintessential large-data problem. Given an information need expressed as a short query consisting of a few terms, the system’s task is to retrieve relevant web objects (web pages, PDF documents, PowerPoint slides, etc.) and present them to the user. How large is the web? It is difficult to compute exactly, but even a conservative estimate would place the size at several tens of billions of pages, totaling hundreds of terabytes (considering text alone). In real-world applications, users demand results quickly from a search engine—query latencies longer than a few hundred milliseconds will try a user’s patience. Fulfilling these requirements is quite an engineering feat, considering the amounts of data involved!

Nearly all retrieval engines for full-text search today rely on a data structure called an inverted index, which given a term provides access to the list of documents that contain the term. In information retrieval parlance, objects to be retrieved are generically called “documents” even though in actuality they may be web pages, PDFs, or even fragments of code. Given a user query, the retrieval engine uses the inverted index to score documents that contain the query terms with respect to some ranking model, taking into account features such as term matches, term proximity, attributes of the terms in the document (e.g., bold, appears in title, etc.), as well as the hyperlink structure of the documents (e.g., PageRank [117], which we’ll discuss in Chapter 5, or related metrics such as HITS [84] and SALSA [88]).

The web search problem decomposes into three components: gathering web content (crawling), construction of the inverted index (indexing) and ranking documents given a query (retrieval). Crawling and indexing share similar characteristics and requirements, but these are very different from retrieval. Gathering web content and building inverted indexes are for the most part offline problems. Both need to be scalable and efficient, but they do not need to operate in real time. Indexing is usually a batch process that runs periodically: the frequency of refreshes and updates is usually dependent on the
design of the crawler. Some sites (e.g., news organizations) update their content quite frequently and need to be visited often; other sites (e.g., government regulations) are relatively static. However, even for rapidly changing sites, it is usually tolerable to have a delay of a few minutes until content is searchable. Furthermore, since the amount of content that changes rapidly is relatively small, running smaller-scale index updates at greater frequencies is usually an adequate solution.\footnote{Leaving aside the problem of searching live data streams such as tweets, which requires different techniques and algorithms.}

Retrieval, on the other hand, is an online problem that demands sub-second response time. Individual users expect low query latencies, but query throughput is equally important since a retrieval engine must usually serve many users concurrently. Furthermore, query loads are highly variable, depending on the time of day, and can exhibit “spikey” behavior due to special circumstances (e.g., a breaking news event triggers a large number of searches on the same topic). On the other hand, resource consumption for the indexing problem is more predictable.

A comprehensive treatment of web search is beyond the scope of this chapter, and even this entire book. Explicitly recognizing this, we mostly focus on the problem of inverted indexing, the task most amenable to solutions in MapReduce. This chapter begins by first providing an overview of web crawling (Section 4.1) and introducing the basic structure of an inverted index (Section 4.2). A baseline inverted indexing algorithm in MapReduce is presented in Section 4.3. We point out a scalability bottleneck in that algorithm, which leads to a revised version presented in Section 4.4. Index compression is discussed in Section 4.5, which fills in missing details on building compact index structures. Since MapReduce is primarily designed for batch-oriented processing, it does not provide an adequate solution for the retrieval problem, an issue we discuss in Section 4.6. The chapter concludes with a summary and pointers to additional readings.

### 4.1 Web Crawling

Before building inverted indexes, we must first acquire the document collection over which these indexes are to be built. In academia and for research purposes, this can be relatively straightforward. Standard collections for information retrieval research are widely available for a variety of genres ranging from blogs to newswire text. For researchers who wish to explore web-scale retrieval, there is the ClueWeb09 collection that contains one billion web pages in ten languages (totaling 25 terabytes) crawled by Carnegie Mellon University in early 2009.\footnote{http://boston.lti.cs.cmu.edu/Data/clueweb09/} Obtaining access to these standard collections is usually as simple as signing an appropriate data license from the distributor of the collection, paying a reasonable fee, and arranging for receipt of the data.\footnote{As an interesting side note, in the 1990s, research collections were distributed via postal mail on CD-ROMs, and later, on DVDs. Electronic distribution became common earlier this decade for collections below a certain size. However, many collections today are so large that...}
For real-world web search, however, one cannot simply assume that the collection is already available. Acquiring web content requires crawling, which is the process of traversing the web by repeatedly following hyperlinks and storing downloaded pages for subsequent processing. Conceptually, the process is quite simple to understand: we start by populating a queue with a “seed” list of pages. The crawler downloads pages in the queue, extracts links from those pages to add to the queue, stores the pages for further processing, and repeats. In fact, rudimentary web crawlers can be written in a few hundred lines of code.

However, effective and efficient web crawling is far more complex. The following lists a number of issues that real-world crawlers must contend with:

- A web crawler must practice good “etiquette” and not overload web servers. For example, it is common practice to wait a fixed amount of time before repeated requests to the same server. In order to respect these constraints while maintaining good throughput, a crawler typically keeps many execution threads running in parallel and maintains many TCP connections (perhaps hundreds) open at the same time.

- Since a crawler has finite bandwidth and resources, it must prioritize the order in which unvisited pages are downloaded. Such decisions must be made online and in an adversarial environment, in the sense that spammers actively create “link farms” and “spider traps” full of spam pages to trick a crawler into overrepresenting content from a particular site.

- Most real-world web crawlers are distributed systems that run on clusters of machines, often geographically distributed. To avoid downloading a page multiple times and to ensure data consistency, the crawler as a whole needs mechanisms for coordination and load-balancing. It also needs to be robust with respect to machine failures, network outages, and errors of various types.

- Web content changes, but with different frequency depending on both the site and the nature of the content. A web crawler needs to learn these update patterns to ensure that content is reasonably current. Getting the right recrawl frequency is tricky: too frequent means wasted resources, but not frequent enough leads to stale content.

- The web is full of duplicate content. Examples include multiple copies of a popular conference paper, mirrors of frequently-accessed sites such as Wikipedia, and newswire content that is often duplicated. The problem is compounded by the fact that most repetitious pages are not exact duplicates but near duplicates (that is, basically the same page but with different ads, navigation bars, etc.) It is desirable during the crawling process to identify near duplicates and select the best exemplar to index.

The only practical method of distribution is shipping hard drives via postal mail.
- The web is multilingual. There is no guarantee that pages in one language only link to pages in the same language. For example, a professor in Asia may maintain her website in the local language, but contain links to publications in English. Furthermore, many pages contain a mix of text in different languages. Since document processing techniques (e.g., tokenization, stemming) differ by language, it is important to identify the (dominant) language on a page.

The above discussion is not meant to be an exhaustive enumeration of issues, but rather to give the reader an appreciation of the complexities involved in this intuitively simple task. For more information, see a recent survey on web crawling [113]. Section 4.7 provides pointers to additional readings.

### 4.2 Inverted Indexes

In its basic form, an inverted index consists of postings lists, one associated with each term that appears in the collection. The structure of an inverted index is illustrated in Figure 4.1. A postings list is comprised of individual postings, each of which consists of a document id and a payload—information about occurrences of the term in the document. The simplest payload is... nothing! For simple boolean retrieval, no additional information is needed in the posting other than the document id; the existence of the posting itself indicates that presence of the term in the document. The most common payload, however, is term frequency \((tf)\), or the number of times the term occurs in the document. More complex payloads include positions of every occurrence of the term in the document (to support phrase queries and document scoring based on term proximity), properties of the term (such as if it occurred in the page title or not, to support document ranking based on notions of importance), or even the results of additional linguistic processing (for example, indicating that the term is part of a place name, to support address searches). In the web context, anchor text information (text associated with hyperlinks from other pages to the page in question) is useful in enriching the representation of document content (e.g., [107]); this information is often stored in the index as well.

In the example shown in Figure 4.1, we see that:

- **term\(_1\)** occurs in \(\{d_1, d_5, d_6, d_{11}, \ldots\}\),
- **term\(_2\)** occurs in \(\{d_{11}, d_{23}, d_{59}, d_{84}, \ldots\}\), and
- **term\(_3\)** occurs in \(\{d_1, d_4, d_{11}, d_{19}, \ldots\}\).

In an actual implementation, we assume that documents can be identified by a unique integer ranging from 1 to \(n\), where \(n\) is the total number of documents. Generally, postings are sorted by document id, although other sort
orders are possible as well. The document ids have no inherent semantic meaning, although assignment of numeric ids to documents need not be arbitrary. For example, pages from the same domain may be consecutively numbered. Or, alternatively, pages that are higher in quality (based, for example, on PageRank values) might be assigned smaller numeric values so that they appear toward the front of a postings list. Either way, an auxiliary data structure is necessary to maintain the mapping from integer document ids to some other more meaningful handle, such as a URL.

Given a query, retrieval involves fetching postings lists associated with query terms and traversing the postings to compute the result set. In the simplest case, boolean retrieval involves set operations (union for boolean OR and intersection for boolean AND) on postings lists, which can be accomplished very efficiently since the postings are sorted by document id. In the general case, however, query–document scores must be computed. Partial document scores are stored in structures called accumulators. At the end (i.e., once all postings have been processed), the top \( k \) documents are then extracted to yield a ranked list of results for the user. Of course, there are many optimization strategies for query evaluation (both approximate and exact) that reduce the number of postings a retrieval engine must examine.

The size of an inverted index varies, depending on the payload stored in each posting. If only term frequency is stored, a well-optimized inverted index can be a tenth of the size of the original document collection. An inverted index that stores positional information would easily be several times larger than one that does not. Generally, it is possible to hold the entire vocabulary (i.e., dictionary of all the terms) in memory, especially with techniques such as front-coding [156]. However, with the exception of well-resourced, commercial web search engines, postings lists are usually too large to store in memory and must be held on disk, usually in compressed form (more details in Section 4.5). Query

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6Google keeps indexes in memory.
Algorithm 4.1 Baseline inverted indexing algorithm

Mappers emit postings keyed by terms, the execution framework groups postings by term, and the reducers write postings lists to disk.

1: **class** Mapper
2: **procedure** Map(docid n, doc d)
3:     \[ H \leftarrow \text{new AssociativeArray} \]
4:     **for all** term \( t \in \text{doc} \) \( d \) \do
5:         \[ H\{t\} \leftarrow H\{t\} + 1 \]
6:     **for all** term \( t \in H \) \do
7:         Emit(term \( t \), posting \( \langle n, H\{t}\rangle \))

1: **class** Reducer
2: **procedure** Reduce(term \( t \), postings \( [\langle n_1, f_1 \rangle, \langle n_2, f_2 \rangle \ldots] \))
3:     \[ P \leftarrow \text{new List} \]
4:     **for all** posting \( \langle a, f \rangle \in \text{postings} \ [\langle n_1, f_1 \rangle, \langle n_2, f_2 \rangle \ldots] \) \do
5:         Append\( (P, \langle a, f \rangle) \)
6:     Sort\( (P) \)
7:     Emit(term \( t \), postings \( P \))

evaluation, therefore, necessarily involves random disk access and “decoding” of the postings. One important aspect of the retrieval problem is to organize disk operations such that random seeks are minimized.

Once again, this brief discussion glosses over many complexities and does a huge injustice to the tremendous amount of research in information retrieval. However, our goal is to provide the reader with an overview of the important issues; Section 4.7 provides references to additional readings.

### 4.3 Inverted Indexing: Baseline Implementation

MapReduce was designed from the very beginning to produce the various data structures involved in web search, including inverted indexes and the web graph. We begin with the basic inverted indexing algorithm shown in Algorithm 4.1.

Input to the mapper consists of document ids (keys) paired with the actual content (values). Individual documents are processed in parallel by the mappers. First, each document is analyzed and broken down into its component terms. The processing pipeline differs depending on the application and type of document, but for web pages typically involves stripping out HTML tags and other elements such as JavaScript code, tokenizing, case folding, removing stopwords (common words such as ‘the’, ‘a’, ‘of’, etc.), and stemming (removing affixes from words so that ‘dogs’ becomes ‘dog’). Once the document has been analyzed, term frequencies are computed by iterating over all the terms and keeping track of counts. Lines 4 and 5 in the pseudo-code reflect the process of computing term frequencies, but hides the details of document processing.
After this histogram has been built, the mapper then iterates over all terms. For each term, a pair consisting of the document id and the term frequency is created. Each pair, denoted by \( \langle n, H\{t\} \rangle \) in the pseudo-code, represents an individual posting. The mapper then emits an intermediate key-value pair with the term as the key and the posting as the value, in line 7 of the mapper pseudo-code. Although as presented here only the term frequency is stored in the posting, this algorithm can be easily augmented to store additional information (e.g., term positions) in the payload.

In the shuffle and sort phase, the MapReduce runtime essentially performs a large, distributed group by of the postings by term. Without any additional effort by the programmer, the execution framework brings together all the postings that belong in the same postings list. This tremendously simplifies the task of the reducer, which simply needs to gather together all the postings and write them to disk. The reducer begins by initializing an empty list and then appends all postings associated with the same key (term) to the list. The postings are then sorted by document id, and the entire postings list is emitted as a value, with the term as the key. Typically, the postings list is first compressed, but we leave this aside for now (see Section 4.4 for more details). The final key-value pairs are written to disk and comprise the inverted index.

Execution of the complete algorithm is illustrated in Figure 4.2 with a toy example consisting of three documents, three mappers, and two reducers. Intermediate key-value pairs (from the mappers) and the final key-value pairs comprising the inverted index (from the reducers) are shown in the boxes with dotted lines. Postings are shown as pairs of boxes, with the document id on the left and the term frequency on the right.

The MapReduce programming model provides a very concise expression of the inverted indexing algorithm. Its implementation is similarly concise: the basic algorithm can be implemented in as few as a couple dozen lines of code in Hadoop (with minimal document processing). Such an implementation can be completed as a week-long programming assignment in a course for advanced undergraduates or first-year graduate students [83, 93]. In a non-MapReduce indexer, a significant fraction of the code is devoted to grouping postings by term, given constraints imposed by memory and disk (e.g., memory capacity is limited, disk seeks are slow, etc.). In MapReduce, the programmer does not need to worry about any of these issues—most of the heavy lifting is performed by the execution framework.
4.4 Inverted Indexing: Revised Implementation

The inverted indexing algorithm presented in the previous section serves as a reasonable baseline. However, there is a significant scalability bottleneck: the algorithm assumes that there is sufficient memory to hold all postings associated with the same term. Since the basic MapReduce execution framework makes no guarantees about the ordering of values associated with the same key, the reducer first buffers all postings (line 5 of the reducer pseudo-code in Algorithm 4.1) and then performs an in-memory sort before writing the postings to disk.\footnote{See similar discussion in Section 3.4: in principle, this need not be an in-memory sort. It is entirely possible to implement a disk-based sort within the reducer.} For efficient retrieval, postings need to be sorted by document id. However, as collections become larger, postings lists grow longer, and at some point in time, reducers will run out of memory.

There is a simple solution to this problem. Since the execution framework guarantees that keys arrive at each reducer in sorted order, one way to overcome
the scalability bottleneck is to let the MapReduce runtime do the sorting for us. Instead of emitting key-value pairs of the following type:

\[(\text{term } t, \text{posting } (\text{docid}, f))\]

We emit intermediate key-value pairs of the type instead:

\[(\text{tuple } \langle t, \text{docid} \rangle, tf)\]

In other words, the key is a tuple containing the term and the document id, while the value is the term frequency. This is exactly the value-to-key conversion design pattern introduced in Section 3.4. With this modification, the programming model ensures that the postings arrive in the correct order. This, combined with the fact that reducers can hold state across multiple keys, allows postings lists to be created with minimal memory usage. As a detail, remember that we must define a custom partitioner to ensure that all tuples with the same term are shuffled to the same reducer.

The revised MapReduce inverted indexing algorithm is shown in Algorithm 4.2. The mapper remains unchanged for the most part, other than differences in the intermediate key-value pairs. The REDUCE method is called for each key (i.e., \((t, n)\)), and by design, there will only be one value associated with each key. For each key-value pair, a posting can be directly added to the postings list. Since the postings are guaranteed to arrive in sorted order by document id, they can be incrementally coded in compressed form—thus ensuring a small memory footprint. Finally, when all postings associated with the same term have been processed (i.e., \(t \neq t_{\text{prev}}\)), the entire postings list is emitted. The final postings list must be written out in the CLOSE method. As with the baseline algorithm, payloads can be easily changed: by simply replacing the intermediate value \(f\) (term frequency) with whatever else is desired (e.g., term positional information).

There is one more detail we must address when building inverted indexes. Since almost all retrieval models take into account document length when computing query–document scores, this information must also be extracted. Although it is straightforward to express this computation as another MapReduce job, this task can actually be folded into the inverted indexing process. When processing the terms in each document, the document length is known, and can be written out as “side data” directly to HDFS. We can take advantage of the ability for a mapper to hold state across the processing of multiple documents in the following manner: an in-memory associative array is created to store document lengths, which is populated as each document is processed.\(^8\) When the mapper finishes processing input records, document lengths are written out to HDFS (i.e., in the CLOSE method). This approach is essentially a variant of the in-mapper combining pattern. Document length data ends up in \(m\) different files, where \(m\) is the number of mappers; these files are then consolidated into a more compact representation. Alternatively, document length information can

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\(^8\)In general, there is no worry about insufficient memory to hold these data.
Algorithm 4.2 Scalable inverted indexing

By applying the value-to-key conversion design pattern, the execution framework is exploited to sort postings so that they arrive sorted by document id in the reducer.

1: class Mapper
2: method Map(docid n, doc d)
3: H ← new AssociativeArray
4: for all term t ∈ doc d do
5: H{t} ← H{t} + 1
6: for all term t ∈ H do
7: Emit(tuple ⟨t, n⟩, tf H{t})

1: class Reducer
2: method Initialize
3: tprev ← ∅
4: P ← new PostingsList
5: method Reduce(tuple ⟨t, n⟩, tf [f])
6: if t ≠ tprev ∧ tprev ≠ ∅ then
7: Emit(term tprev, postings P)
8: P.Reset()
9: P.Add((⟨n, f⟩))
10: tprev ← t
11: method Close
12: Emit(term t, postings P)

be emitted in special key-value pairs by the mapper. One must then write a custom partitioner so that these special key-value pairs are shuffled to a single reducer, which will be responsible for writing out the length data separate from the postings lists.

4.5 Index Compression

We return to the question of how postings are actually compressed and stored on disk. This chapter devotes a substantial amount of space to this topic because index compression is one of the main differences between a “toy” indexer and one that works on real-world collections. Otherwise, MapReduce inverted indexing algorithms are pretty straightforward.

Let us consider the canonical case where each posting consists of a document id and the term frequency. A naïve implementation might represent the first as a 32-bit integer\(^9\) and the second as a 16-bit integer. Thus, a postings list might be encoded as follows:

\(^9\)However, note that \(2^{32} - 1\) is “only” 4,294,967,295, which is much less than even the most conservative estimate of the size of the web.
where each posting is represented by a pair in parentheses. Note that all brackets, parentheses, and commas are only included to enhance readability; in reality the postings would be represented as a long stream of integers. This naïve implementation would require six bytes per posting. Using this scheme, the entire inverted index would be about as large as the collection itself. Fortunately, we can do significantly better.

The first trick is to encode differences between document ids as opposed to the document ids themselves. Since the postings are sorted by document ids, the differences (called \(d\)-gaps) must be positive integers greater than zero. The above postings list, represented with \(d\)-gaps, would be:

\[
[(5, 2), (2, 3), (5, 1), (37, 1), (2, 2), \ldots]
\]

Of course, we must actually encode the first document id. We haven’t lost any information, since the original document ids can be easily reconstructed from the \(d\)-gaps. However, it’s not obvious that we’ve reduced the space requirements either, since the largest possible \(d\)-gap is one less than the number of documents in the collection.

This is where the second trick comes in, which is to represent the \(d\)-gaps in a way such that it takes less space for smaller numbers. Similarly, we want to apply the same techniques to compress the term frequencies, since for the most part they are also small values. But to understand how this is done, we need to take a slight detour into compression techniques, particularly for coding integers.

Compression, in general, can be characterized as either lossless or lossy: it’s fairly obvious that lossless compression is required in this context. To start, it is important to understand that all compression techniques represent a time–space tradeoff. That is, we reduce the amount of space on disk necessary to store data, but at the cost of extra processor cycles that must be spent coding and decoding data. Therefore, it is possible that compression reduces size but also slows processing. However, if the two factors are properly balanced (i.e., decoding speed can keep up with disk bandwidth), we can achieve the best of both worlds: smaller and faster.

### Byte-Aligned and Word-Aligned Codes

In most programming languages, an integer is encoded in four bytes and holds a value between 0 and \(2^{32} - 1\), inclusive. We limit our discussion to unsigned integers, since \(d\)-gaps are always positive (and greater than zero). This means that 1 and 4,294,967,295 both occupy four bytes. Obviously, encoding \(d\)-gaps this way doesn’t yield any reductions in size.

A simple approach to compression is to only use as many bytes as is necessary to represent the integer. This is known as variable-length integer coding (varInt for short) and accomplished by using the high order bit of every byte
as the continuation bit, which is set to one in the last byte and zero elsewhere. As a result, we have 7 bits per byte for coding the value, which means that $0 \leq n < 2^7$ can be expressed with 1 byte, $2^7 \leq n < 2^{14}$ with 2 bytes, $2^{14} \leq n < 2^{21}$ with 3, and $2^{21} \leq n < 2^{28}$ with 4 bytes. This scheme can be extended to code arbitrarily-large integers (i.e., beyond 4 bytes). As a concrete example, the two numbers:

127, 128

would be coded as such:

1 1111111, 0 0000001 1 0000000

The above code contains two code words, the first consisting of 1 byte, and the second consisting of 2 bytes. Of course, the comma and the spaces are there only for readability. Variable-length integers are byte-aligned because the code words always fall along byte boundaries. As a result, there is never any ambiguity about where one code word ends and the next begins. However, the downside of varInt coding is that decoding involves lots of bit operations (masks, shifts). Furthermore, the continuation bit sometimes results in frequent branch mispredicts (depending on the actual distribution of $d$-gaps), which slows down processing.

A variant of the varInt scheme was described by Jeff Dean in a keynote talk at the WSDM 2009 conference.\(^\text{10}\) The insight is to code groups of four integers at a time. Each group begins with a prefix byte, divided into four 2-bit values that specify the byte length of each of the following integers. For example, the following prefix byte:

00.00.01.10

indicates that the following four integers are one byte, one byte, two bytes, and three bytes, respectively. Therefore, each group of four integers would consume anywhere between 5 and 17 bytes. A simple lookup table based on the prefix byte directs the decoder on how to process subsequent bytes to recover the coded integers. The advantage of this group varInt coding scheme is that values can be decoded with fewer branch mispredicts and bitwise operations. Experiments reported by Dean suggest that decoding integers with this scheme is more than twice as fast as the basic varInt scheme.

In most architectures, accessing entire machine words is more efficient than fetching all its bytes separately. Therefore, it makes sense to store postings in increments of 16-bit, 32-bit, or 64-bit machine words. Anh and Moffat [8] presented several word-aligned coding methods, one of which is called Simple-9, based on 32-bit words. In this coding scheme, four bits in each 32-bit word are reserved as a selector. The remaining 28 bits are used to code actual integer values. Now, there are a variety of ways these 28 bits can be divided

\(^{10}\) [http://research.google.com/people/jeff/WSDM09-keynote.pdf](http://research.google.com/people/jeff/WSDM09-keynote.pdf)
to code one or more integers: 28 bits can be used to code one 28-bit integer, two 14-bit integers, three 9-bit integers (with one bit unused), etc., all the way up to twenty-eight 1-bit integers. In fact, there are nine different ways the 28 bits can be divided into equal parts (hence the name of the technique), some with leftover unused bits. This is stored in the selector bits. Therefore, decoding involves reading a 32-bit word, examining the selector to see how the remaining 28 bits are packed, and then appropriately decoding each integer. Coding works in the opposite way: the algorithm scans ahead to see how many integers can be squeezed into 28 bits, packs those integers, and sets the selector bits appropriately.

Bit-Aligned Codes

The advantage of byte-aligned and word-aligned codes is that they can be coded and decoded quickly. The downside, however, is that they must consume multiples of eight bits, even when fewer bits might suffice (the Simple-9 scheme gets around this by packing multiple integers into a 32-bit word, but even then, bits are often wasted). In bit-aligned codes, on the other hand, code words can occupy any number of bits, meaning that boundaries can fall anywhere. In practice, coding and decoding bit-aligned codes require processing bytes and appropriately shifting or masking bits (usually more involved than varInt and group varInt coding).

One additional challenge with bit-aligned codes is that we need a mechanism to delimit code words, i.e., tell where the last ends and the next begins, since there are no byte boundaries to guide us. To address this issue, most bit-aligned codes are so-called prefix codes (confusingly, they are also called prefix-free codes), in which no valid code word is a prefix of any other valid code word. For example, coding \(0 \leq x < 3\) with \{0, 1, 01\} is not a valid prefix code, since 0 is a prefix of 01, and so we can’t tell if 01 is two code words or one. On the other hand, \{00, 01, 1\} is a valid prefix code, such that a sequence of bits:

\[
0001101001010100
\]

can be unambiguously segmented into:

\[
00 \ 01 \ 01 \ 00 \ 01 \ 01 \ 00
\]

and decoded without any additional delimiters.

One of the simplest prefix codes is the unary code. An integer \(x > 0\) is coded as \(x - 1\) one bits followed by a zero bit. Note that unary codes do not allow the representation of zero, which is fine since \(d\)-gaps and term frequencies should never be zero.\(^{11}\) As an example, 4 in unary code is 1110. With unary code we can code \(x\) in \(x\) bits, which although economical for small values, becomes inefficient for even moderately large values. Unary codes are rarely

\(^{11}\)As a note, some sources describe slightly different formulations of the same coding scheme. Here, we adopt the conventions used in the classic IR text Managing Gigabytes [156].
used by themselves, but form a component of other coding schemes. Unary codes of the first ten positive integers are shown in Figure 4.3.

Elias $\gamma$ code is an efficient coding scheme that is widely used in practice. An integer $x > 0$ is broken into two components, $1 + \left\lfloor \log_2 x \right\rfloor$ ($= n$, the length), which is coded in unary code, and $x - 2^{\left\lfloor \log_2 x \right\rfloor}$ ($= r$, the remainder), which is in binary. The unary component $n$ specifies the number of bits required to code $x$, and the binary component codes the remainder $r$ in $n - 1$ bits. As an example, consider $x = 10$: $1 + \left\lfloor \log_2 10 \right\rfloor = 4$, which is 1110. The binary component codes $x - 2^3 = 2$ in $4 - 1 = 3$ bits, which is 010. Putting both together, we arrive at 1110:010. The extra colon is inserted only for readability; it’s not part of the final code, of course.

Working in reverse, it is easy to unambiguously decode a bit stream of $\gamma$ codes: First, we read a unary code $c_u$, which is a prefix code. This tells us that the binary portion is written in $c_u - 1$ bits, which we then read as $c_b$. We can then reconstruct $x$ as $2^{c_u-1} + c_b$. For $x < 16$, $\gamma$ codes occupy less than a full byte, which makes them more compact than variable-length integer codes. Since term frequencies for the most part are relatively small, $\gamma$ codes make sense for them and can yield substantial space savings. For reference, the $\gamma$ codes of the first ten positive integers are shown in Figure 4.3. A variation on $\gamma$ code is $\delta$ code, where the $n$ portion of the $\gamma$ code is coded in $\gamma$ code itself (as opposed to unary code). For smaller values, $\gamma$ codes are more compact, but for larger values, $\delta$ codes take less space.

Unary and $\gamma$ codes are parameterless, but even better compression can be achieved with parameterized codes. A good example of this is Golomb code. For some parameter $b$, an integer $x > 0$ is coded in two parts: first, we compute $q = \left\lfloor (x - 1)/b \right\rfloor$ and code $q + 1$ in unary; then, we code the remainder

<table>
<thead>
<tr>
<th>$x$</th>
<th>unary</th>
<th>$\gamma$</th>
<th>Golomb ($b = 5$)</th>
<th>Golomb ($b = 10$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0</td>
<td>0:000</td>
<td>0:000</td>
</tr>
<tr>
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<td>0:01</td>
<td>0:01</td>
<td>0:01</td>
</tr>
<tr>
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<td>0:10</td>
<td>0:10</td>
<td>0:10</td>
</tr>
<tr>
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<td>0:110</td>
<td>0:110</td>
<td>0:110</td>
</tr>
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</tr>
<tr>
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<td>10:00</td>
<td>10:00</td>
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</tr>
<tr>
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<td>10:01</td>
<td>10:01</td>
<td>10:01</td>
</tr>
<tr>
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<tr>
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<td>1111111110</td>
<td>10:111</td>
<td>10:111</td>
<td>10:111</td>
</tr>
</tbody>
</table>

Figure 4.3: The first ten positive integers in unary, $\gamma$, and Golomb ($b = 5, 10$) codes.

Note that $\left\lfloor x \right\rfloor$ is the floor function, which maps $x$ to the largest integer not greater than $x$, so, e.g., $\left\lfloor 3.8 \right\rfloor = 3$. This is the default behavior in many programming languages when casting from a floating-point type to an integer type.
\[ r = x - qb - 1 \] in truncated binary. This is accomplished as follows: if \( b \) is a power of two, then truncated binary is exactly the same as normal binary, requiring \( \log_2 b \) bits. Otherwise, we code the first \( 2^{\lfloor \log_2 b \rfloor + 1} - b \) values of \( r \) in \( \lfloor \log_2 b \rfloor \) bits and code the rest of the values of \( r \) by coding \( r + 2^{\lfloor \log_2 b \rfloor + 1} - b \) in ordinary binary representation using \( \lfloor \log_2 b \rfloor + 1 \) bits. In this case, the \( r \) is coded in either \( \lfloor \log_2 b \rfloor \) or \( \lfloor \log_2 b \rfloor + 1 \) bits, and unlike ordinary binary coding, truncated binary codes are prefix codes. As an example, if \( b = 5 \), then \( r \) can take the values \( \{0, 1, 2, 3, 4\} \), which would be coded with the following code words: \( \{00, 01, 10, 110, 111\} \). For reference, Golomb codes of the first ten positive integers are shown in Figure 4.3 for \( b = 5 \) and \( b = 10 \). A special case of Golomb code is worth noting: if \( b \) is a power of two, then coding and decoding can be handled more efficiently (needing only bit shifts and bit masks, as opposed to multiplication and division). These are known as Rice codes.

Researchers have shown that Golomb compression works well for \( d \)-gaps, and is optimal with the following parameter setting:

\[
b \approx 0.69 \times \frac{df}{N} \tag{4.1}
\]

where \( df \) is the document frequency of the term, and \( N \) is the number of documents in the collection.\(^{13}\)

Putting everything together, one popular approach for postings compression is to represent \( d \)-gaps with Golomb codes and term frequencies with \( \gamma \) codes \([156, 162]\). If positional information is desired, we can use the same trick to code differences between term positions using \( \gamma \) codes.

### Postings Compression

Having completed our slight detour into integer compression techniques, we can now return to the scalable inverted indexing algorithm shown in Algorithm 4.2 and discuss how postings lists can be properly compressed. As we can see from the previous section, there is a wide range of choices that represent different tradeoffs between compression ratio and decoding speed. Actual performance also depends on characteristics of the collection, which, among other factors, determine the distribution of \( d \)-gaps. Büttcher et al. \([30]\) recently compared the performance of various compression techniques on coding document ids. In terms of the amount of compression that can be obtained (measured in bits per \textit{docid}), Golomb and Rice codes performed the best, followed by \( \gamma \) codes, Simple-9, varInt, and group varInt (the least space efficient). In terms of raw decoding speed, the order was almost the reverse: group varInt was the fastest, followed by varInt.\(^{14}\) Simple-9 was substantially slower, and the bit-aligned

\(^{13}\)For details as to why this is the case, we refer the reader elsewhere \([156]\), but here’s the intuition: under reasonable assumptions, the appearance of postings can be modeled as a sequence of independent Bernoulli trials, which implies a certain distribution of \( d \)-gaps. From this we can derive an optimal setting of \( b \).

\(^{14}\)However, this study found less speed difference between group varInt and basic varInt than Dean’s analysis, presumably due to the different distribution of \( d \)-gaps in the collections they were examining.
codes were even slower than that. Within the bit-aligned codes, Rice codes were the fastest, followed by $\gamma$, with Golomb codes being the slowest (about ten times slower than group varInt).

Let us discuss what modifications are necessary to our inverted indexing algorithm if we were to adopt Golomb compression for $d$-gaps and represent term frequencies with $\gamma$ codes. Note that this represents a space-efficient encoding, at the cost of slower decoding compared to alternatives. Whether or not this is actually a worthwhile tradeoff in practice is not important here: use of Golomb codes serves a pedagogical purpose, to illustrate how one might set compression parameters.

Coding term frequencies with $\gamma$ codes is easy since they are parameterless. Compressing $d$-gaps with Golomb codes, however, is a bit tricky, since two parameters are required: the size of the document collection and the number of postings for a particular postings list (i.e., the document frequency, or $df$). The first is easy to obtain and can be passed into the reducer as a constant. The $df$ of a term, however, is not known until all the postings have been processed—and unfortunately, the parameter must be known before any posting is coded. At first glance, this seems like a chicken-and-egg problem. A two-pass solution that involves first buffering the postings (in memory) would suffer from the memory bottleneck we’ve been trying to avoid in the first place.

To get around this problem, we need to somehow inform the reducer of a term’s $df$ before any of its postings arrive. This can be solved with the order inversion design pattern introduced in Section 3.3 to compute relative frequencies. The solution is to have the mapper emit special keys of the form $\langle t, * \rangle$ to communicate partial document frequencies. That is, inside the mapper, in addition to emitting intermediate key-value pairs of the following form:

$$(\text{tuple } \langle t, \text{docid} \rangle, \text{tf } f)$$

we also emit special intermediate key-value pairs like this:

$$(\text{tuple } \langle t, * \rangle, \text{df } e)$$

to keep track of document frequencies associated with each term. In practice, we can accomplish this by applying the in-mapper combining design pattern (see Section 3.1). The mapper holds an in-memory associative array that keeps track of how many documents a term has been observed in (i.e., the local document frequency of the term for the subset of documents processed by the mapper). Once the mapper has processed all input records, special keys of the form $\langle t, * \rangle$ are emitted with the partial $df$ as the value.

To ensure that these special keys arrive first, we define the sort order of the tuple so that the special symbol $*$ precedes all documents (part of the order inversion design pattern). Thus, for each term, the reducer will first encounter the $\langle t, * \rangle$ key, associated with a list of values representing partial $df$ values originating from each mapper. Summing all these partial contributions will yield the term’s $df$, which can then be used to set the Golomb compression
parameter $b$. This allows the postings to be incrementally compressed as they are encountered in the reducer—memory bottlenecks are eliminated since we do not need to buffer postings in memory.

Once again, the order inversion design pattern comes to the rescue. Recall that the pattern is useful when a reducer needs to access the result of a computation (e.g., an aggregate statistic) before it encounters the data necessary to produce that computation. For computing relative frequencies, that bit of information was the marginal. In this case, it’s the document frequency.

4.6 What About Retrieval?

Thus far, we have briefly discussed web crawling and focused mostly on MapReduce algorithms for inverted indexing. What about retrieval? It should be fairly obvious that MapReduce, which was designed for large batch operations, is a poor solution for retrieval. Since users demand sub-second response times, every aspect of retrieval must be optimized for low latency, which is exactly the opposite tradeoff made in MapReduce. Recall the basic retrieval problem: we must look up postings lists corresponding to query terms, systematically traverse those postings lists to compute query–document scores, and then return the top $k$ results to the user. Looking up postings implies random disk seeks, since for the most part postings are too large to fit into memory (leaving aside caching and other special cases for now). Unfortunately, random access is not a forte of the distributed file system underlying MapReduce—such operations require multiple round-trip network exchanges (and associated latencies). In HDFS, a client must first obtain the location of the desired data block from the namenode before the appropriate datanode can be contacted for the actual data. Of course, access will typically require a random disk seek on the datanode itself.

It should be fairly obvious that serving the search needs of a large number of users, each of whom demand sub-second response times, is beyond the capabilities of any single machine. The only solution is to distribute retrieval across a large number of machines, which necessitates breaking up the index in some manner. There are two main partitioning strategies for distributed retrieval: document partitioning and term partitioning. Under document partitioning, the entire collection is broken up into multiple smaller sub-collections, each of which is assigned to a server. In other words, each server holds the complete index for a subset of the entire collection. This corresponds to partitioning vertically in Figure 4.4. With term partitioning, on the other hand, each server is responsible for a subset of the terms for the entire collection. That is, a server holds the postings for all documents in the collection for a subset of terms. This corresponds to partitioning horizontally in Figure 4.4.

Document and term partitioning require different retrieval strategies and represent different tradeoffs. Retrieval under document partitioning involves a query broker, which forwards the user’s query to all partition servers, merges partial results from each, and then returns the final results to the user. With
Figure 4.4: Term–document matrix for a toy collection (nine documents, nine terms) illustrating different partitioning strategies: partitioning vertically (1, 2, 3) corresponds to document partitioning, whereas partitioning horizontally (a, b, c) corresponds to term partitioning.

Under this architecture, searching the entire collection requires that the query be processed by every partition server. However, since each partition operates independently and traverses postings in parallel, document partitioning typically yields shorter query latencies (compared to a single monolithic index with much longer postings lists).

Retrieval under term partitioning, on the other hand, requires a very different strategy. Suppose the user’s query $Q$ contains three terms, $q_1$, $q_2$, and $q_3$. Under the pipelined query evaluation strategy, the broker begins by forwarding the query to the server that holds the postings for $q_1$ (usually the least frequent term). The server traverses the appropriate postings list and computes partial query–document scores, stored in the accumulators. The accumulators are then passed to the server that holds the postings associated with $q_2$ for additional processing, and then to the server for $q_3$, before final results are passed back to the broker and returned to the user. Although this query evaluation strategy may not substantially reduce the latency of any particular query, it can theoretically increase a system’s throughput due to the far smaller number of total disk seeks required for each user query (compared to document partitioning).
However, load-balancing is tricky in a pipelined term-partitioned architecture due to skew in the distribution of query terms, which can create “hot spots” on servers that hold the postings for frequently-occurring query terms.

In general, studies have shown that document partitioning is a better strategy overall [109], and this is the strategy adopted by Google [16]. Furthermore, it is known that Google maintains its indexes in memory (although this is certainly not the common case for search engines in general). One key advantage of document partitioning is that result quality degrades gracefully with machine failures. Partition servers that are offline will simply fail to deliver results for their subsets of the collection. With sufficient partitions, users might not even be aware that documents are missing. For most queries, the web contains more relevant documents than any user has time to digest: users of course care about getting relevant documents (sometimes, they are happy with a single relevant document), but they are generally less discriminating when it comes to which relevant documents appear in their results (out of the set of all relevant documents). Note that partitions may be unavailable due to reasons other than machine failure: cycling through different partitions is a very simple and non-disruptive strategy for index updates.

Working in a document-partitioned architecture, there are a variety of approaches to dividing up the web into smaller pieces. Proper partitioning of the collection can address one major weakness of this architecture, which is that every partition server is involved in every user query. Along one dimension, it is desirable to partition by document quality using one or more classifiers; see [124] for a recent survey on web page classification. Partitioning by document quality supports a multi-phase search strategy: the system examines partitions containing high quality documents first, and only backs off to partitions containing lower quality documents if necessary. This reduces the number of servers that need to be contacted for a user query. Along an orthogonal dimension, it is desirable to partition documents by content (perhaps also guided by the distribution of user queries from logs), so that each partition is “well separated” from the others in terms of topical coverage. This also reduces the number of machines that need to be involved in serving a user’s query: the broker can direct queries only to the partitions that are likely to contain relevant documents, as opposed to forwarding the user query to all the partitions.

On a large-scale, reliability of service is provided by replication, both in terms of multiple machines serving the same partition within a single datacenter, but also replication across geographically-distributed datacenters. This creates at least two query routing problems: since it makes sense to serve clients from the closest datacenter, a service must route queries to the appropriate location. Within a single datacenter, the system needs to properly balance load across replicas.

There are two final components of real-world search engines that are worth discussing. First, recall that postings only store document ids. Therefore, raw retrieval results consist of a ranked list of semantically meaningless document ids. It is typically the responsibility of document servers, functionally distinct from the partition servers holding the indexes, to generate meaningful output.
for user presentation. Abstractly, a document server takes as input a query and a document id, and computes an appropriate result entry, typically comprising the title and URL of the page, a snippet of the source document showing the user’s query terms in context, and additional metadata about the document. Second, query evaluation can benefit immensely from caching, of individual postings (assuming that the index is not already in memory) and even results of entire queries [13]. This is made possible by the Zipfian distribution of queries, with very frequent queries at the head of the distribution dominating the total number of queries. Search engines take advantage of this with cache servers, which are functionally distinct from all of the components discussed above.

4.7 Summary and Additional Readings

Web search is a complex problem that breaks down into three conceptually-distinct components. First, the documents collection must be gathered (by crawling the web). Next, inverted indexes and other auxiliary data structures must be built from the documents. Both of these can be considered offline problems. Finally, index structures must be accessed and processed in response to user queries to generate search results. This last task is an online problem that demands both low latency and high throughput.

This chapter primarily focused on building inverted indexes, the problem most suitable for MapReduce. After all, inverted indexing is nothing but a very large distributed sort and group by operation! We began with a baseline implementation of an inverted indexing algorithm, but quickly noticed a scalability bottleneck that stemmed from having to buffer postings in memory. Application of the value-to-key conversion design pattern (Section 3.4) addressed the issue by offloading the task of sorting postings by document id to the MapReduce execution framework. We also surveyed various techniques for integer compression, which yield postings lists that are both more compact and faster to process. As a specific example, one could use Golomb codes for compressing \( d \)-gaps and \( \gamma \) codes for term frequencies. We showed how the order inversion design pattern introduced in Section 3.3 for computing relative frequencies can be used to properly set compression parameters.

**Additional Readings.** Our brief discussion of web search glosses over many complexities and does a huge injustice to the tremendous amount of research in information retrieval. Here, however, we provide a few entry points into the literature. A survey article by Zobel and Moffat [162] is an excellent starting point on indexing and retrieval algorithms. Another by Baeza-Yates et al. [11] overviews many important issues in distributed retrieval. A keynote talk at the WSDM 2009 conference by Jeff Dean revealed a lot of information about the evolution of the Google search architecture.\(^\text{15}\) Finally, a number of general information retrieval textbooks have been recently published [101, 42, 30]. Of

\(^{15}\)\url{http://research.google.com/people/jeff/WSDM09-keynote.pdf}
these three, the one by Büttcher et al. [30] is noteworthy in having detailed experimental evaluations that compare the performance (both effectiveness and efficiency) of a wide range of algorithms and techniques. While outdated in many other respects, the textbook *Managing Gigabytes* [156] remains an excellent source for index compression techniques. Finally, ACM SIGIR is an annual conference and the most prestigious venue for academic information retrieval research; proceedings from those events are perhaps the best starting point for those wishing to keep abreast of publicly-documented developments in the field.