Big Data Infrastructure

CS 489/698 Big Data Infrastructure (Winter 2016)

Week 9: Data Mining (3/4)

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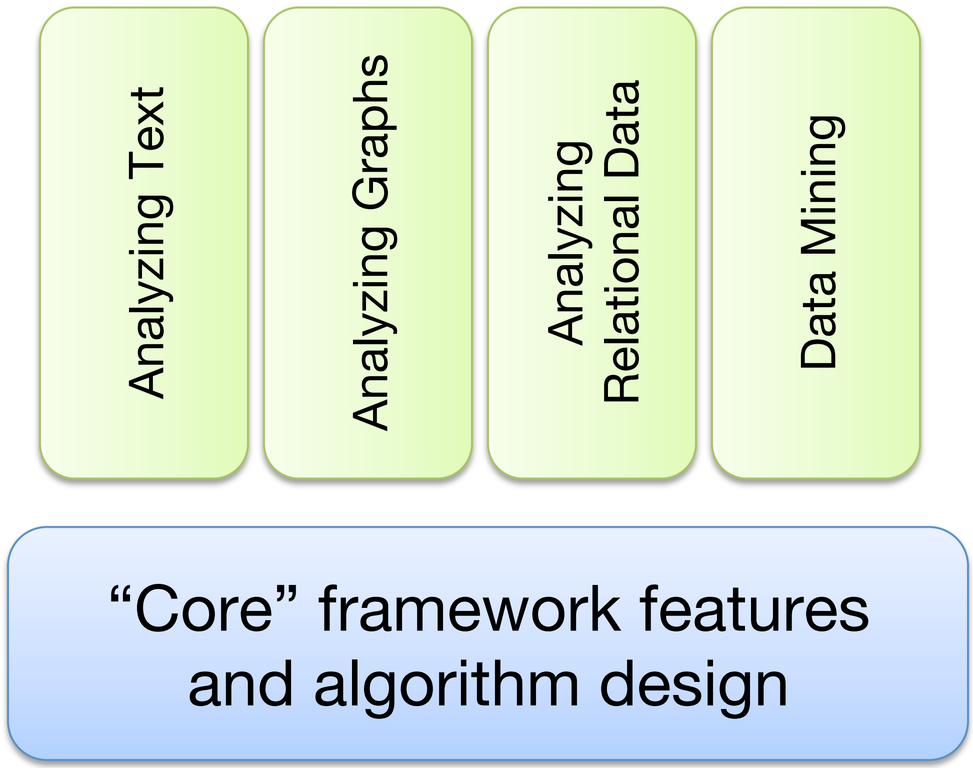
These slides are available at http://lintool.github.io/bigdata-2016w/

Structure of the Course



What's the Problem?

- Finding similar items with respect to some distance metric
- Two variants of the problem:
  - Offline: extract all similar pairs of objects from a large collection
  - Online: is this object similar to something I've seen before?

Literature Note

- Many communities have tackled similar problems:
  - Theoretical computer science
  - Information retrieval
  - Data mining
  - Databases
  - …
- Issues
  - Slightly different terminology
  - Results not easy to compare

Four Steps

- Specify distance metric
  - Jaccard, Euclidean, cosine, etc.
- Compute representation
  - Shingling, tf.idf, etc.
- "Project"
  - Minhash, random projections, etc.
- Extract
  - Bucketing, sliding windows, etc.

Distances

Source: www.flickr.com/photos/thiagoalmeida/250190676/

Distance Metrics

1. Non-negativity:

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) \geq 0$$

2. Identity:

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) = 0 \iff \mathrm{x} = \mathrm{y}$$

3. Symmetry:

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) = \mathrm{d}(\mathrm{y}, \mathrm{x})$$

4. Triangle Inequality

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) \leq \mathrm{d}(\mathrm{x}, \mathrm{z}) + \mathrm{d}(\mathrm{z}, \mathrm{y})$$

Distance: Jaccard

- Given two sets A, B
- Jaccard similarity:

$$\mathrm{J}(A,B) = \frac{|A \cap B|}{|A \cup B|}$$

$$\mathrm{d}(A,B) = 1 - \mathrm{J}(A,B)$$

Distance: Norms

- Given: $$\mathrm{x} = [x_1, x_2, \ldots x_n]$$ $$\mathrm{y} = [y_1, y_2, \ldots y_n]$$
- Euclidean distance (L_2-norm)

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) = \sqrt{\sum_{i=0}^{n} (x_i - y_i)^2}$$

- Manhattan distance (L_1-norm)

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) = \sum_{i=0}^{n} |x_i - y_i|$$

- L_r-norm

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) = \left[\sum_{i=0}^{n} |x_i - y_i|^r\right]^{1/r}$$

Distance: Cosine

- Given: $$\mathrm{x} = [x_1, x_2, \ldots x_n]$$ $$\mathrm{y} = [y_1, y_2, \ldots y_n]$$

- Idea: measure distance between the vectors

$$\cos\theta = \frac{\mathrm{x} \cdot \mathrm{y}}{|\mathrm{x}||\mathrm{y}|}$$

- Thus:

$$\mathrm{sim}(\mathrm{x}, \mathrm{y}) = \frac{\sum_{i=0}^{n} x_i y_i}{\sqrt{\sum_{i=0}^{n} x_i^2}\sqrt{\sum_{i=0}^{n} y_i^2}}$$

$$\mathrm{d}(\mathrm{x}, \mathrm{y}) = 1 - \mathrm{sim}(\mathrm{x}, \mathrm{y})$$

Distance: Hamming

- Given two bit vectors
- Hamming distance: number of elements which differ

Representations

Representations: Text

- Unigrams (i.e., words)
- Shingles = *n*-grams
  - At the word level
  - At the character level
- Feature weights
  - boolean
  - tf.idf
  - BM25
  - …

Representations: Beyond Text

- For recommender systems:
  - Items as features for users
  - Users as features for items
- For graphs:
  - Adjacency lists as features for vertices
- With log data:
  - Behaviors (clicks) as features

Minhash

Source: www.flickr.com/photos/rheinitz/6158837748/

Near-Duplicate Detection of Webpages

- What's the source of the problem?
  - Mirror pages (legit)
  - Spam farms (non-legit)
  - Additional complications (e.g., nav bars)
- Naïve algorithm:
  - Compute cryptographic hash for webpage (e.g., MD5)
  - Insert hash values into a big hash table
  - Compute hash for new webpage: collision implies duplicate
- What's the issue?
- Intuition:
  - Hash function needs to be tolerant of minor differences
  - High similarity implies higher probability of hash collision

Minhash

- Naïve approach: N^2 comparisons: Can we do better?
- Seminal algorithm for near-duplicate detection of webpages
  - Used by AltaVista
  - For details see Broder et al. (1997)
- Setup:
  - Documents (HTML pages) represented by shingles (*n*-grams)
  - Jaccard similarity: dups are pairs with high similarity

Preliminaries: Representation

- Sets:
  - $A = \{e_1, e_3, e_7\}$
  - $B = \{e_3, e_5, e_7\}$
- Can be equivalently expressed as matrices:

Element	A	B
e_1	1	0
e_2	0	0
e_3	1	1
e_4	0	0
e_5	0	1
e_6	0	0
e_7	1	1

Preliminaries: Jaccard

Element	A	B
e_1	1	0
e_2	0	0
e_3	1	1
e_4	0	0
e_5	0	1
e_6	0	0
e_7	1	1

Let:

M_{00} = # rows where both elements are 0

M_{11} = # rows where both elements are 1

M_{01} = # rows where A=0, B=1

M_{10} = # rows where A=1, B=0

$$\mathrm{J}(A, B) = \frac{M_{11}}{M_{01} + M_{10} + M_{11}}$$

Minhash

- Computing minhash
  - Start with the matrix representation of the set
  - Randomly permute the rows of the matrix
  - minhash is the first row with a "one"
- Example:

$h(A) = e_3$ $h(B) = e_5$

Element	A	B
e_1	1	0
e_2	0	0
e_3	1	1
e_4	0	0
e_5	0	1
e_6	0	0
e_7	1	1

Element	A	B
e_6	0	0
e_2	0	0
e_5	0	1
e_3	1	1
e_7	1	1
e_4	0	0
e_1	1	0

Minhash and Jaccard

Element	A	B	
e_6	0	0	M_{00}
e_2	0	0	M_{00}
e_5	0	1	M_{01}
e_3	1	1	M_{11}
e_7	1	1	M_{11}
e_4	0	0	M_{00}
e_1	1	0	M_{10}

$$P[h(A) = h(B)] = \mathrm{J}(A, B)$$

$$\frac{M_{11}}{M_{01} + M_{10} + M_{11}} \qquad \frac{M_{11}}{M_{01} + M_{10} + M_{11}}$$

Woah!

To Permute or Not to Permute?

- Permutations are expensive
- Interpret the hash value as the permutation
- Only need to keep track of the minimum hash value
  - Can keep track of multiple minhash values at once

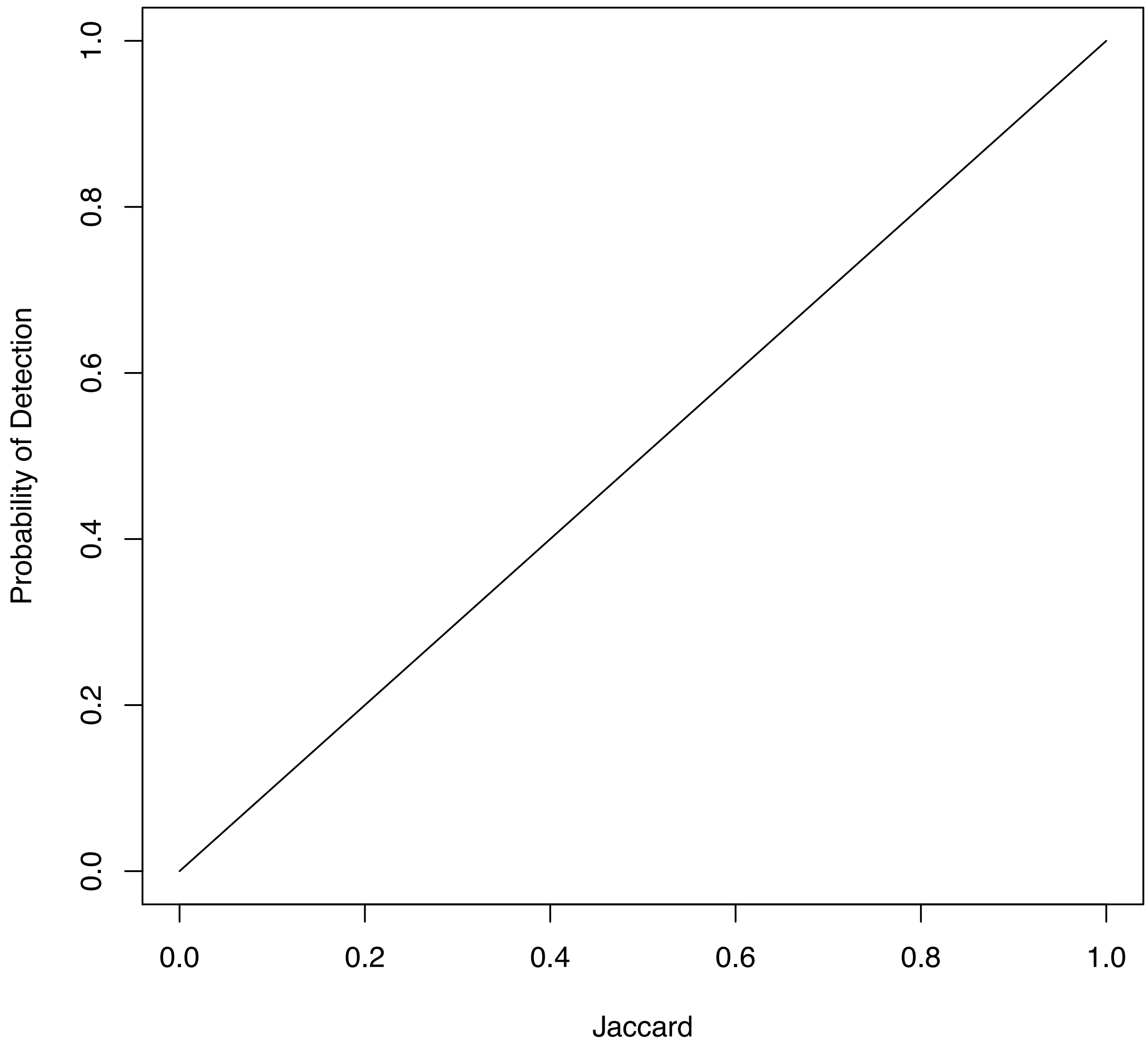
Extracting Similar Pairs

- Task: discover all pairs with similarity greater than *s*
- Naïve approach: N^2 comparisons: Can we do better?
- Tradeoffs:
  - False positives: discovered pairs that have similarity less than *s*
  - False negatives: pairs with similarity greater than *s* not discovered

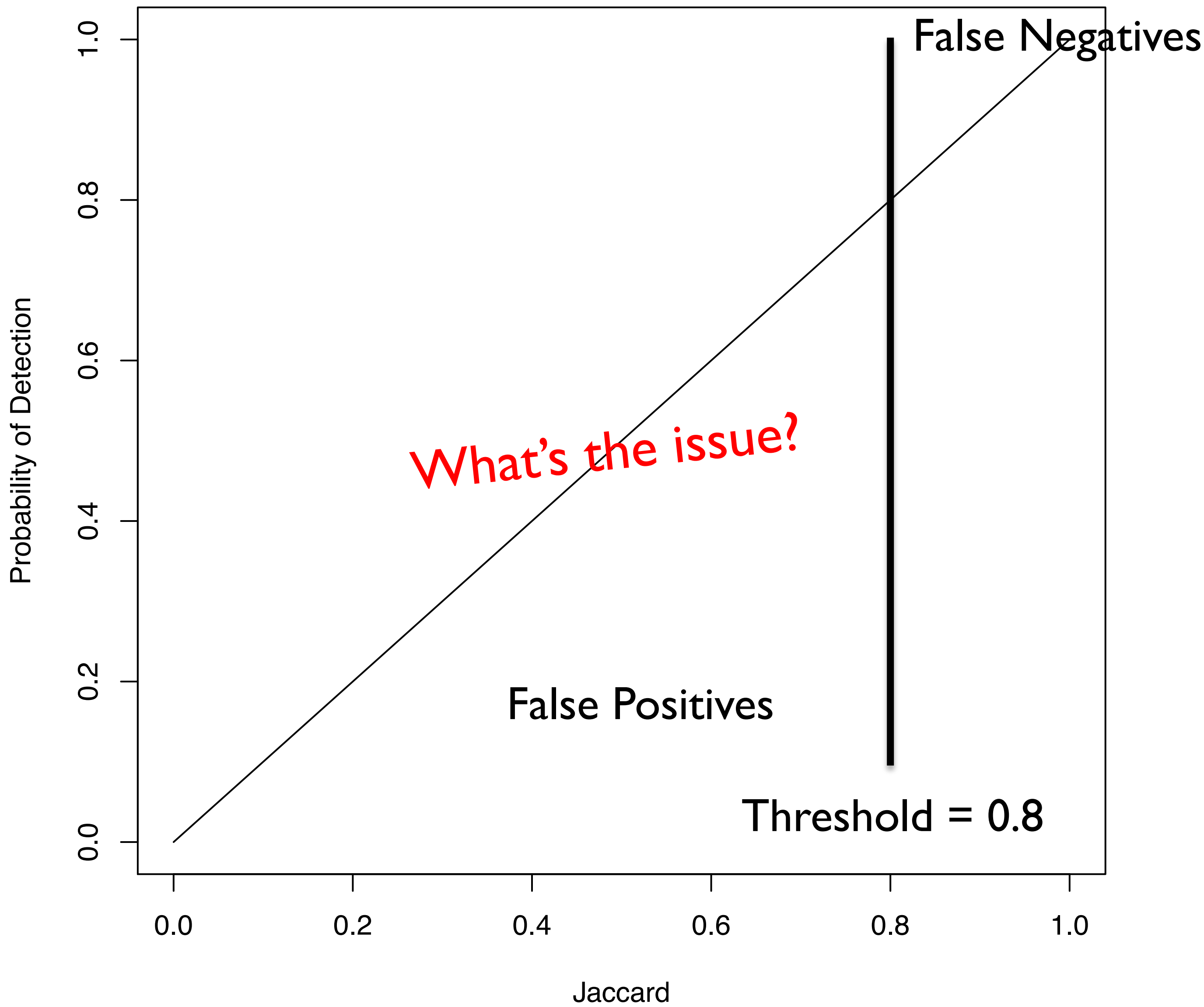
The errors (and costs) are asymmetric!

Extracting Similar Pairs (LSH)

- We know: $P[h(A) = h(B)] = \mathrm{J}(A, B)$
- Task: discover all pairs with similarity greater than *s*
- Algorithm:
  - For each object, compute its minhash value
  - Group objects by their hash values
  - Output all pairs within each group
- Analysis:
  - If J(A,B) = *s*, then probability we detect it is *s*



1.0
0.8
0.6
0.4
0.2
0.0
Probability of Detection
0.0
0.2
0.4
0.6
0.8
1.0
Jaccard


False Negatives
What's the issue?
False Positives
Threshold = 0.8
Probability of Detection
Jaccard
0.0
0.2
0.4
0.6
0.8
1.0

2 Minhash Signatures

- Task: discover all pairs with similarity greater than s
- Algorithm:
  - For each object, compute 2 minhash values and concatenate together into a signature
  - Group objects by their signatures
  - Output all pairs within each group
- Analysis:
  - If $J(A,B) = s$, then probability we detect it is s^2

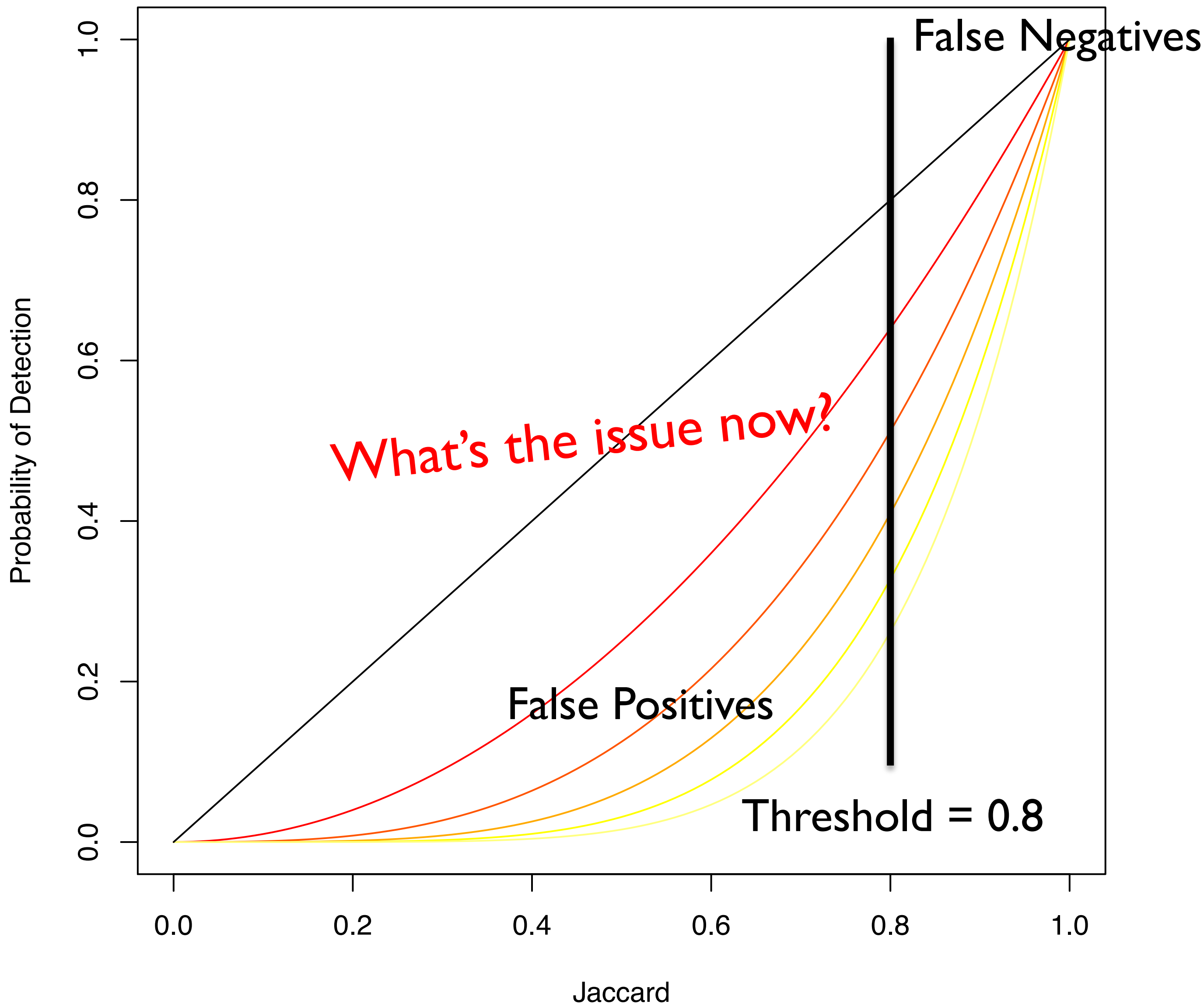
3 Minhash Signatures

- Task: discover all pairs with similarity greater than s
- Algorithm:
  - For each object, compute 3 minhash values and concatenate together into a signature
  - Group objects by their signatures
  - Output all pairs within each group
- Analysis:
  - If $J(A,B) = s$, then probability we detect it is s^3

k Minhash Signatures

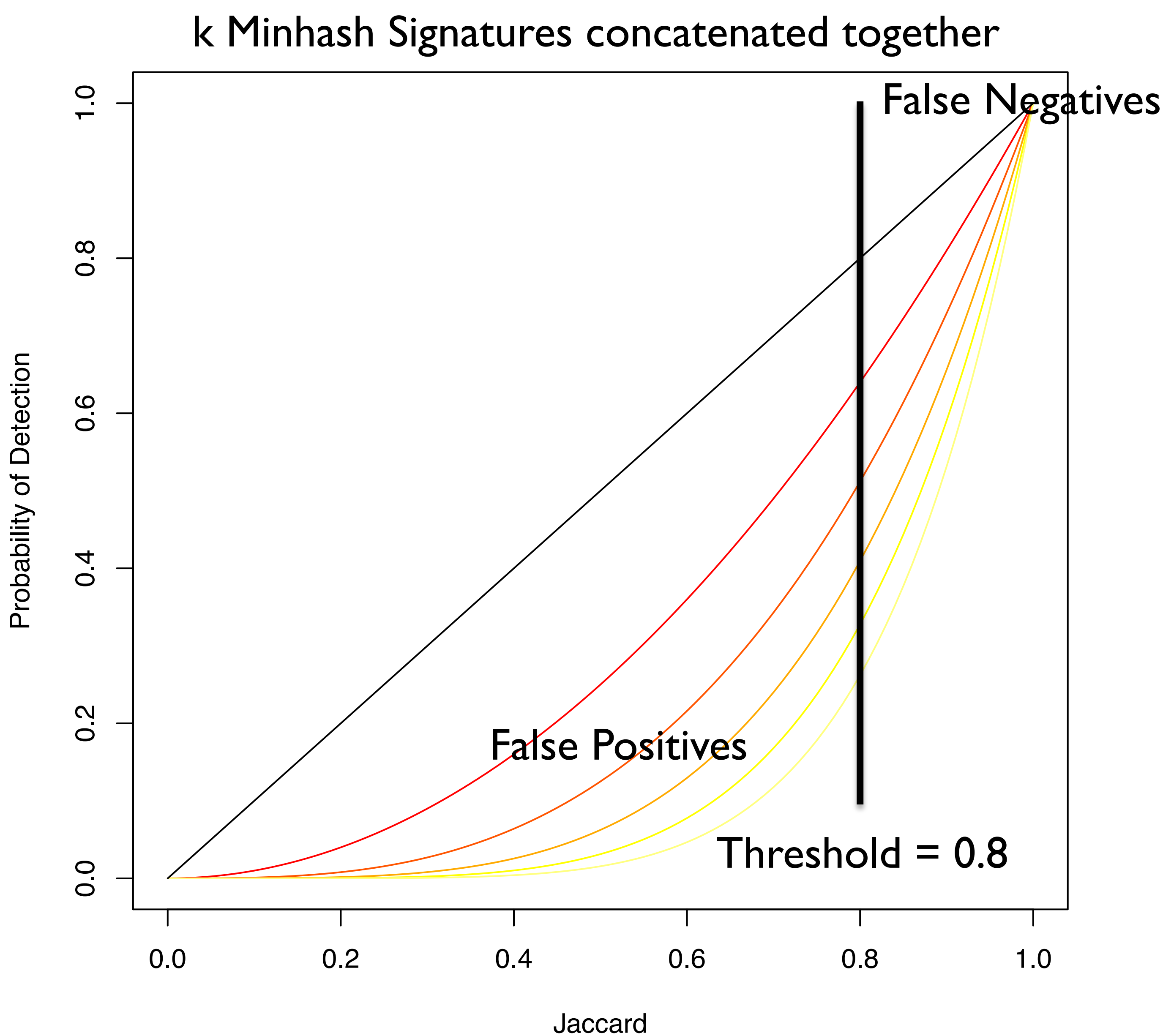
- Task: discover all pairs with similarity greater than s
- Algorithm:
  - For each object, compute k minhash values and concatenate together into a signature
  - Group objects by their signatures
  - Output all pairs within each group
- Analysis:
  - If $J(A,B) = s$, then probability we detect it is s^k

k Minhash Signatures concatenated together



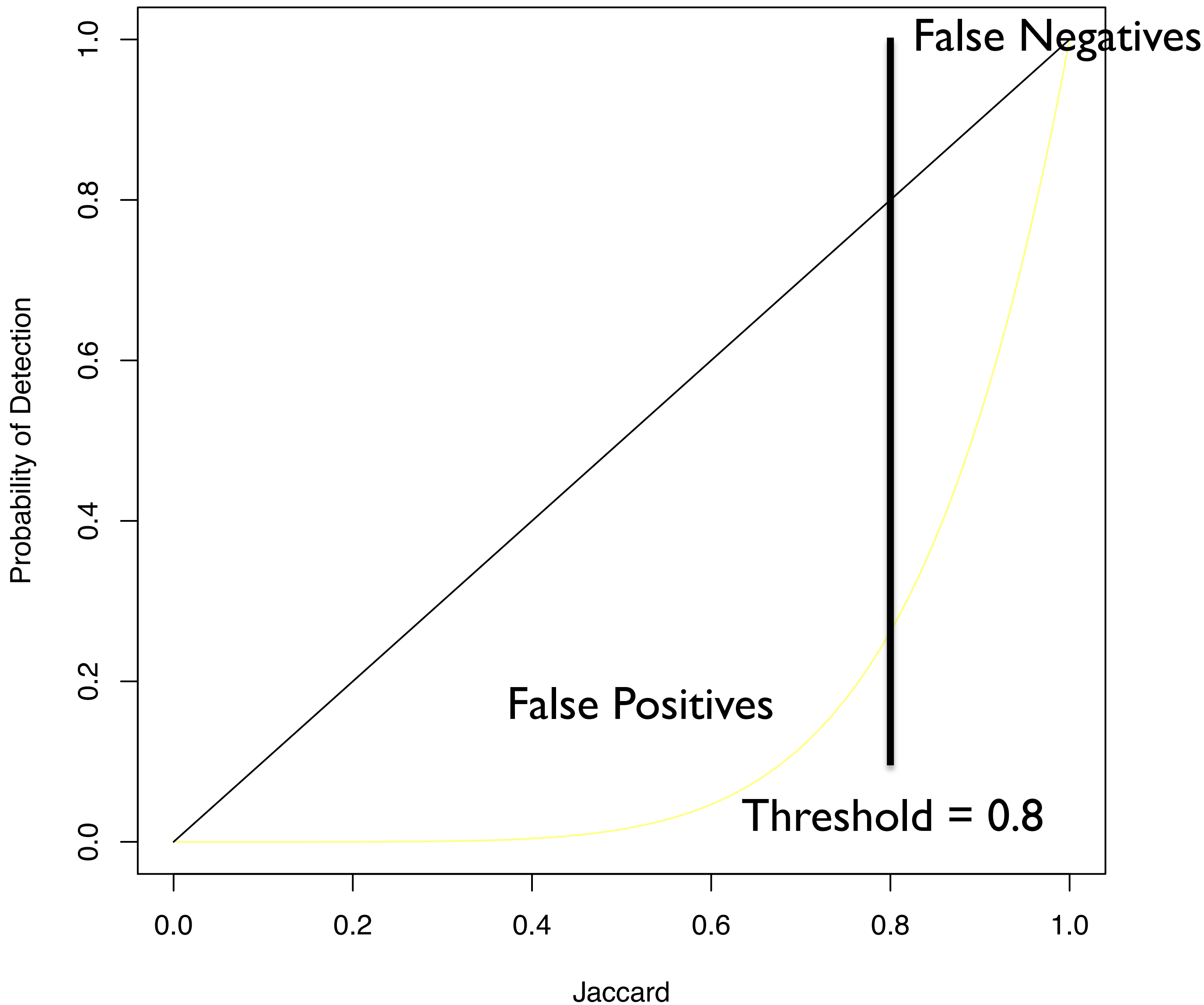
n different *k* Minhash Signatures

- Task: discover all pairs with similarity greater than *s*
- Algorithm:
  - For each object, compute *n* sets *k* minhash values
  - For each set, concatenate *k* minhash values together
  - Within each set:
    - Group objects by their signatures
    - Output all pairs within each group
  - De-dup pairs
- Analysis:
  - If J(A,B) = s, P(none of the n collide) = $(1 - s^k)^n$
  - If J(A,B) = s, then probability we detect it is $1 - (1 - s^k)^n$

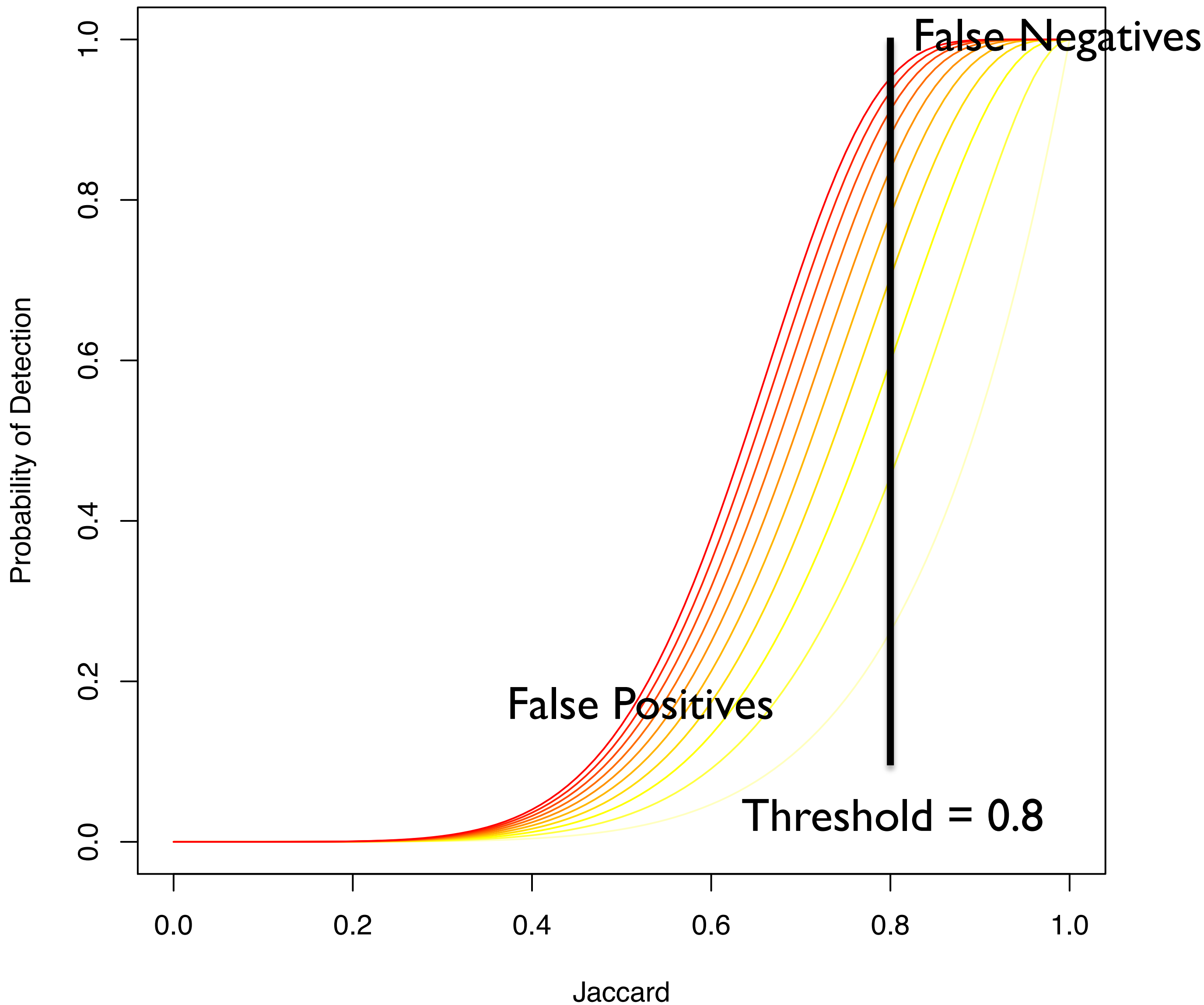


k Minhash Signatures concatenated together
False Negatives
False Positives
Threshold = 0.8
Probability of Detection
Jaccard
0.0
0.2
0.4
0.6
0.8
1.0

6 Minhash Signatures concatenated together



n different sets of 6 Minhash Signatures



n different *k* Minhash Signatures

- Example: J(A,B) = 0.8, 10 sets of 6 minhash signatures
  - P(*k* minhash signatures match) = $(0.8)^6 = 0.262$
  - P(*k* minhash signature doesn't match in any of the 10 sets) = $(1 – (0.8)^6)^{10} = 0.0478$
  - Thus, we should find $1 – (1 – (0.8)^6)^{10} = 0.952$ of all similar pairs
- Example: J(A,B) = 0.4, 10 sets of 6 minhash signatures
  - P(*k* minhash signatures match) = $(0.4)^6 = 0.0041$
  - P(*k* minhash signature doesn't match in any of the 10 sets) = $(1 – (0.4)^6)^{10} = 0.9598$
  - Thus, we should find $1 – (1 – 0.262144)^{10} = 0.040$ of all similar pairs

n different *k* Minhash Signatures

s	$1 - (1 - s^6)^{10}$
0.2	0.0006
0.3	0.0073
0.4	0.040
0.5	0.146
0.6	0.380
0.7	0.714
0.8	0.952
0.9	0.999

What's the issue?

Practical Notes

- Common implementation:
  - Generate M minhash values, select k of them n times
  - Reduces amount of hash computations needed
- Determining “authoritative” version is non-trivial

MapReduce/Spark Implementation

- Map over objects:
  - Generate *M* minhash values, select *k* of them *n* times
  - Each draw yields a signature, emit:
    key = (*p*, signature), where *p* = [1 … *n*]
    value = object id
- Shuffle/sort:
- Reduce:
  - Receive all object ids with same (*n*, signature), emit clusters
- Second pass to de-dup and group clusters
- (Optional) Third pass to eliminate false positives

Offline Extraction vs. Online Querying

- Batch formulation of the problem:
  - Discover all pairs with similarity greater than s
  - Useful for post-hoc batch processing of web crawl
- Online formulation of the problem:
  - Given new webpage, is it similar to one I've seen before?
  - Useful for incremental web crawl processing

Online Similarity Querying

- Preparing the existing collection:
  - For each object, compute *n* sets of *k* minhash values
  - For each set, concatenate *k* minhash values together
  - Keep each signature in hash table (in memory)
  - Note: can parallelize across multiple machines
- Querying and updating:
  - For new webpage, compute signatures and check for collisions
  - Collisions imply duplicate (determine which version to keep)
  - Update hash tables

Random Projections

Source: www.flickr.com/photos/roj/4179478228/

Limitations of Minhash

- Minhash is great for near-duplicate detection
  - Set high threshold for Jaccard similarity
- Limitations:
  - Jaccard similarity only
  - Set-based representation, no way to assign weights to features
- Random projections:
  - Works with arbitrary vectors using cosine similarity
  - Same basic idea, but details differ
  - Slower but more accurate: no free lunch!

Random Projection Hashing

- Generate a random vector *r* of unit length
  - Draw from univariate Gaussian for each component
  - Normalize length
- Define:

$$h_r(\mathrm{u}) = \begin{cases} 1 & \text{if } \mathrm{r} \cdot \mathrm{u} \geq 0 \\ 0 & \text{if } \mathrm{r} \cdot \mathrm{u} < 0 \end{cases}$$

- Physical intuition?

RP Hash Collisions

- It can be shown that:

$$P[h_r(\mathrm{u}) = h_r(\mathrm{v})] = 1 - \frac{\theta(\mathrm{u},\mathrm{v})}{\pi}$$

  - Proof in (Goemans and Williamson, 1995)

- Thus:

$$\cos(\theta(\mathrm{u},\mathrm{v})) = \cos((1 - P[h_r(\mathrm{u}) = h_r(\mathrm{v})])\pi)$$

- Physical intuition?

Random Projection Signature

- Given *D* random vectors:

$$[\mathrm{r}_1, \mathrm{r}_2, \mathrm{r}_3, \ldots \mathrm{r}_D]$$

- Convert each object into a *D* bit signature

$$\mathrm{u} \rightarrow [h_{r_1}(\mathrm{u}), h_{r_2}(\mathrm{u}), h_{r_3}(\mathrm{u}), \ldots h_{r_D}(\mathrm{u})]$$

  - Since:

$$\cos(\theta(\mathrm{u}, \mathrm{v})) = \cos((1 - P[h_r(\mathrm{u}) = h_r(\mathrm{v})])\pi)$$

  - We can derive:

$$\cos(\theta(\mathrm{u}, \mathrm{v})) = \cos\left(\frac{\mathrm{hamming}(\mathrm{s}_\mathrm{u}, \mathrm{s}_\mathrm{v})}{D} \cdot \pi\right)$$

- Thus: similarity boils down to comparison of hamming distances between signatures

One-RP Signature

- Task: discover all pairs with cosine similarity greater than s
- Algorithm:
  - Compute D-bit RP signature for every object
  - Take first bit, bucket objects into two sets
  - Perform brute force pairwise (hamming distance) comparison in each bucket, retain those below hamming distance threshold
- Analysis:
  - Probability we will discover all pairs:*

$$1 - \frac{\cos^{-1}(s)}{\pi}$$

  - Efficiency:

$$N^2 \qquad \text{vs.} \qquad 2\left(\frac{N}{2}\right)^2$$

* Note, this is actually a simplification: see Ture et al. (SIGIR 2011) for details.

Two-RP Signature

- Task: discover all pairs with cosine similarity greater than s
- Algorithm:
  - Compute D-bit RP signature for every object
  - Take first two bits, bucket objects into four sets
  - Perform brute force pairwise (hamming distance) comparison in each bucket, retain those below hamming distance threshold
- Analysis:
  - Probability we will discover all pairs:

$$\left[1 - \frac{\cos^{-1}(s)}{\pi}\right]^2$$

  - Efficiency:

$$N^2 \qquad \text{vs.} \qquad 4\left(\frac{N}{4}\right)^2$$

k-RP Signature

- Task: discover all pairs with cosine similarity greater than *s*
- Algorithm:
  - Compute *D*-bit RP signature for every object
  - Take first *k* bits, bucket objects into 2^k sets
  - Perform brute force pairwise (hamming distance) comparison in each bucket, retain those below hamming distance threshold
- Analysis:
  - Probability we will discover all pairs:

$$\left[1 - \frac{\cos^{-1}(s)}{\pi}\right]^k$$

  - Efficiency:

$$N^2 \qquad \text{vs.} \qquad 2^k \left(\frac{N}{2^k}\right)^2$$

m Sets of *k*-RP Signature

- Task: discover all pairs with cosine similarity greater than s
- Algorithm:
  - Compute D-bit RP signature for every object
  - Choose m sets of k bits
  - For each set, use k selected bits to partition objects into 2^k sets
  - Perform brute force pairwise (hamming distance) comparison in each bucket (of each set), retain those below hamming distance threshold
- Analysis:
  - Probability we will discover all pairs:

$$1 - \left[1 - \left[1 - \frac{\cos^{-1}(s)}{\pi}\right]^k\right]^m$$

  - Efficiency: $N^2 \quad \text{vs.} \quad m \cdot 2^k \left(\frac{N}{2^k}\right)^2$

MapReduce/Spark Implementation

- Map over objects:
  - Compute D-bit RP signature for every object
  - Choose m sets of k bits and use to bucket; for each, emit:
    key = (p, k bits), where p = [1 … m]
    value = (object id, rest of signature bits)
- Shuffle/sort:
- Reduce:
  - Receive (p, k bits)
  - Perform brute force pairwise (hamming distance) comparison for each key, retain those below hamming distance threshold
- Second pass to de-dup and group clusters
- (Optional) Third pass to eliminate false positives

Online Querying

- Preprocessing:
  - Compute D-bit RP signature for every object
  - Choose m sets of k bits and use to bucket
  - Store signatures in memory (across multiple machines)
- Querying
  - Compute D-bit signature of query object, choose m sets of k bits in same way
  - Perform brute-force scan of correct bucket (in parallel)

Additional Issues to Consider

- Emphasis on recall, not precision
- Two sources of error:
  - From LSH
  - From using hamming distance as proxy for cosine similarity
- Load imbalance
- Parameter tuning

"Sliding Window" Algorithm

- Compute *D*-bit RP signature for every object
- For each object, permute bit signature *m* times
- For each permutation, sort bit signatures
  - Apply sliding window of width *B* over sorted
  - Compute hamming distances of bit signatures within window

MapReduce Implementation

- Mapper:
  - Process each individual object in parallel
  - Load in random vectors as side data
  - Compute bit signature
  - Permute m times, for each emit:
    key = (p, signature), where $p = [\ 1 \ldots m\ ]$
    value = object id
- Reduce
  - Keep FIFO queue of B bit signatures
  - For each newly-encountered bit signature, compute hamming distance wrt all bit signatures in queue
  - Add new bit signature to end of queue, displacing oldest

Four Steps to Finding Similar Items

- Specify distance metric
  - Jaccard, Euclidean, cosine, etc.
- Compute representation
  - Shingling, tf.idf, etc.
- "Project"
  - Minhash, random projections, etc.
- Extract
  - Bucketing, sliding windows, etc.

Source: Wikipedia (Japanese rock garden)