UNIVERSITY OF WATERLOO

Big Data Infrastructure

CS 489/698 Big Data Infrastructure (Winter 2016)

Week 5: Analyzing Graphs (1/2)

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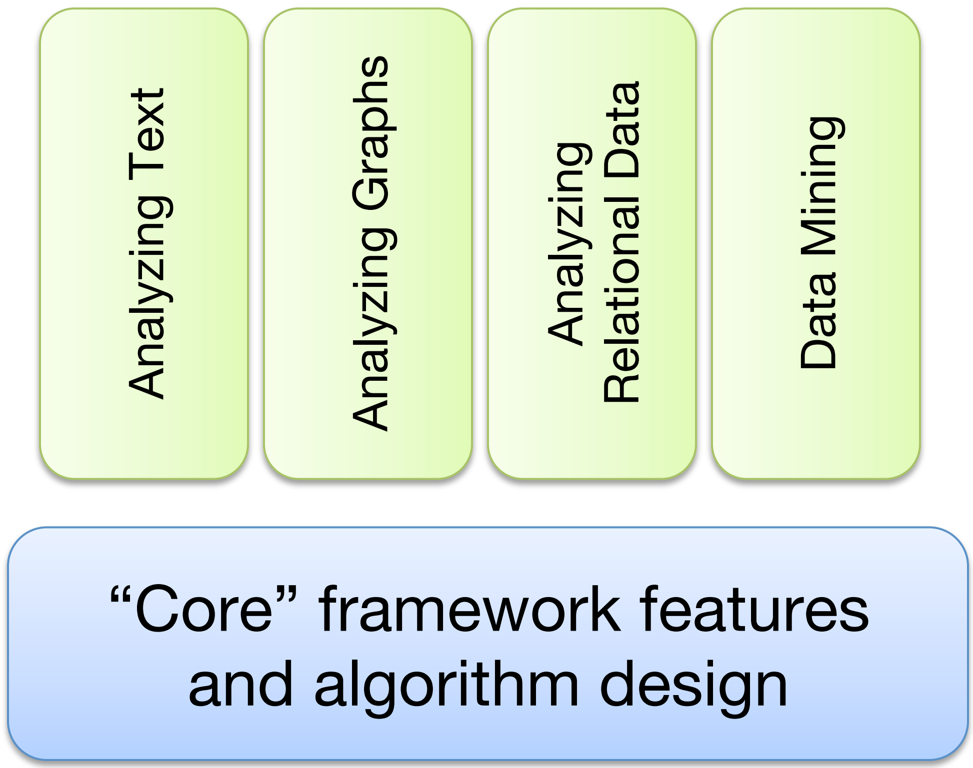
These slides are available at http://lintool.github.io/bigdata-2016w/

Structure of the Course



What's a graph?

- G = (V,E), where
  - V represents the set of vertices (nodes)
  - E represents the set of edges (links)
  - Both vertices and edges may contain additional information
- Different types of graphs:
  - Directed vs. undirected edges
  - Presence or absence of cycles
- Graphs are everywhere:
  - Hyperlink structure of the web
  - Physical structure of computers on the Internet
  - Interstate highway system
  - Social networks

Source: Wikipedia (Königsberg)


KONINGSBERGA

Source: Wikipedia (Kaliningrad)

Some Graph Problems

- Finding shortest paths
  - Routing Internet traffic and UPS trucks
- Finding minimum spanning trees
  - Telco laying down fiber
- Finding Max Flow
  - Airline scheduling
- Identify "special" nodes and communities
  - Breaking up terrorist cells, spread of avian flu
- Bipartite matching
  - Monster.com, Match.com
- And of course... PageRank

What makes graphs hard?

- Irregular structure
- Irregular data access patterns
- Iterations

Graphs and MapReduce (and Spark)

- A large class of graph algorithms involve:
  - Performing computations at each node: based on node features, edge features, and local link structure
  - Propagating computations: "traversing" the graph
- Key questions:
  - How do you represent graph data in MapReduce (and Spark)?
  - How do you traverse a graph in MapReduce (and Spark)?

Representing Graphs

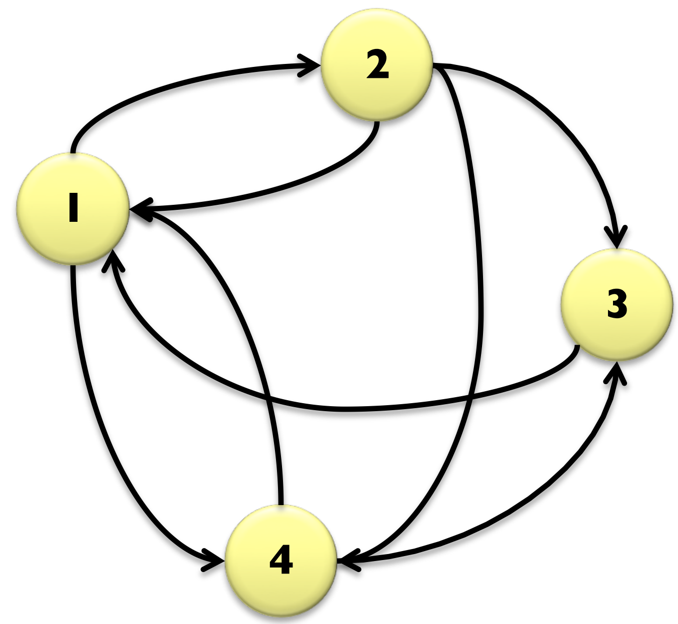
- G = (V, E)
- Three common representations
  - Adjacency matrix
  - Adjacency list
  - Edge lists

Adjacency Matrices

Represent a graph as an n x n square matrix M

- $n = |V|$
- $M_{ij} = 1$ means a link from node i to j

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0



Adjacency Matrices: Critique

- Advantages:
  - Amenable to mathematical manipulation
  - Iteration over rows and columns corresponds to computations on outlinks and inlinks
- Disadvantages:
  - Lots of zeros for sparse matrices
  - Lots of wasted space

Adjacency Lists

Take adjacency matrices… and throw away all the zeros

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

→

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

Wait, where have we seen this before?

Adjacency Lists: Critique

- Advantages:
  - Much more compact representation
  - Easy to compute over outlinks
- Disadvantages:
  - Much more difficult to compute over inlinks

Edge Lists

Explicitly enumerate all edges

	1	2	3	4
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

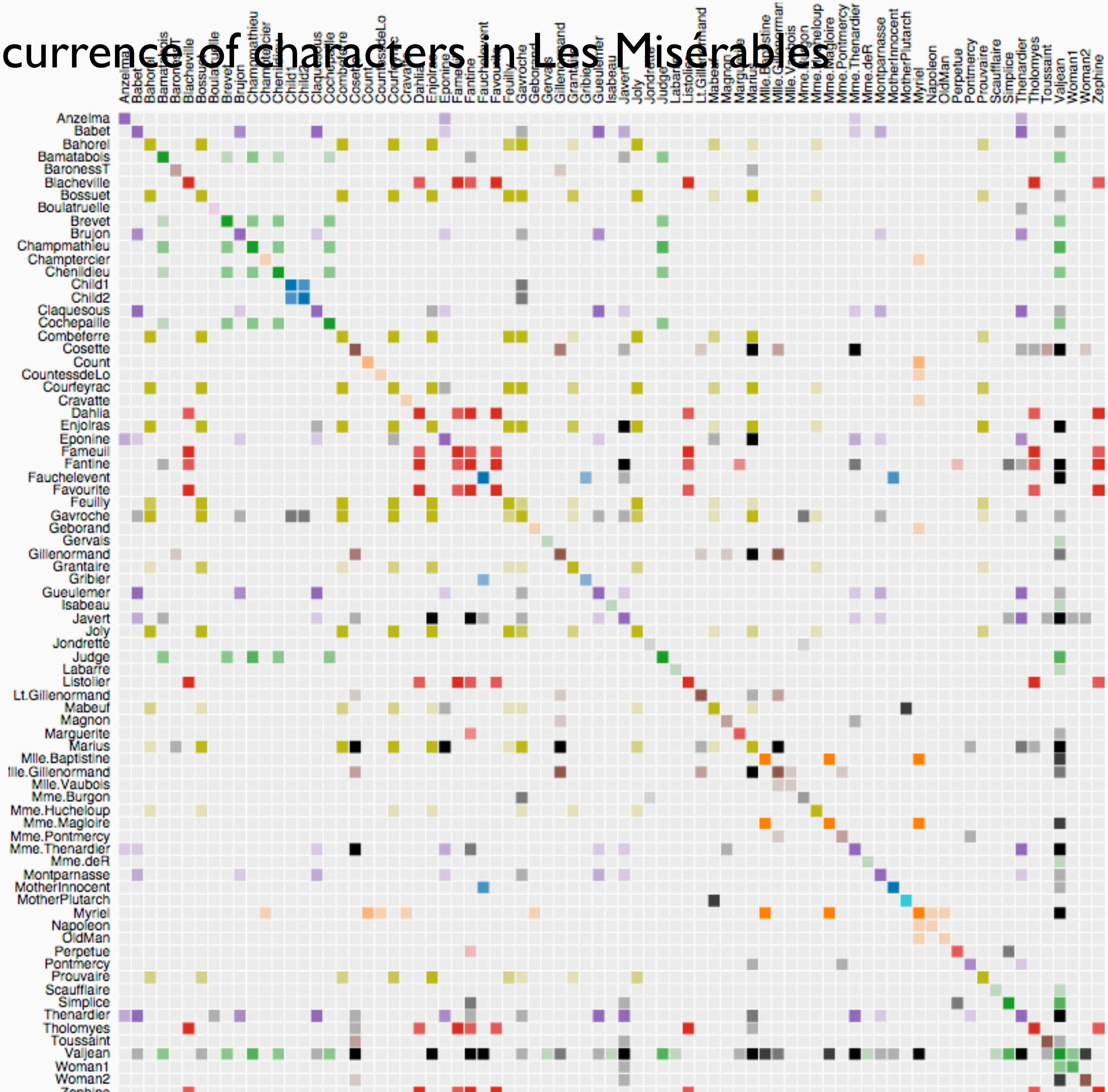
$\Rightarrow$

(1, 2)
(1, 4)
(2, 1)
(2, 3)
(2, 4)
(3, 1)
(4, 1)
(4, 3)

Edge Lists: Critique

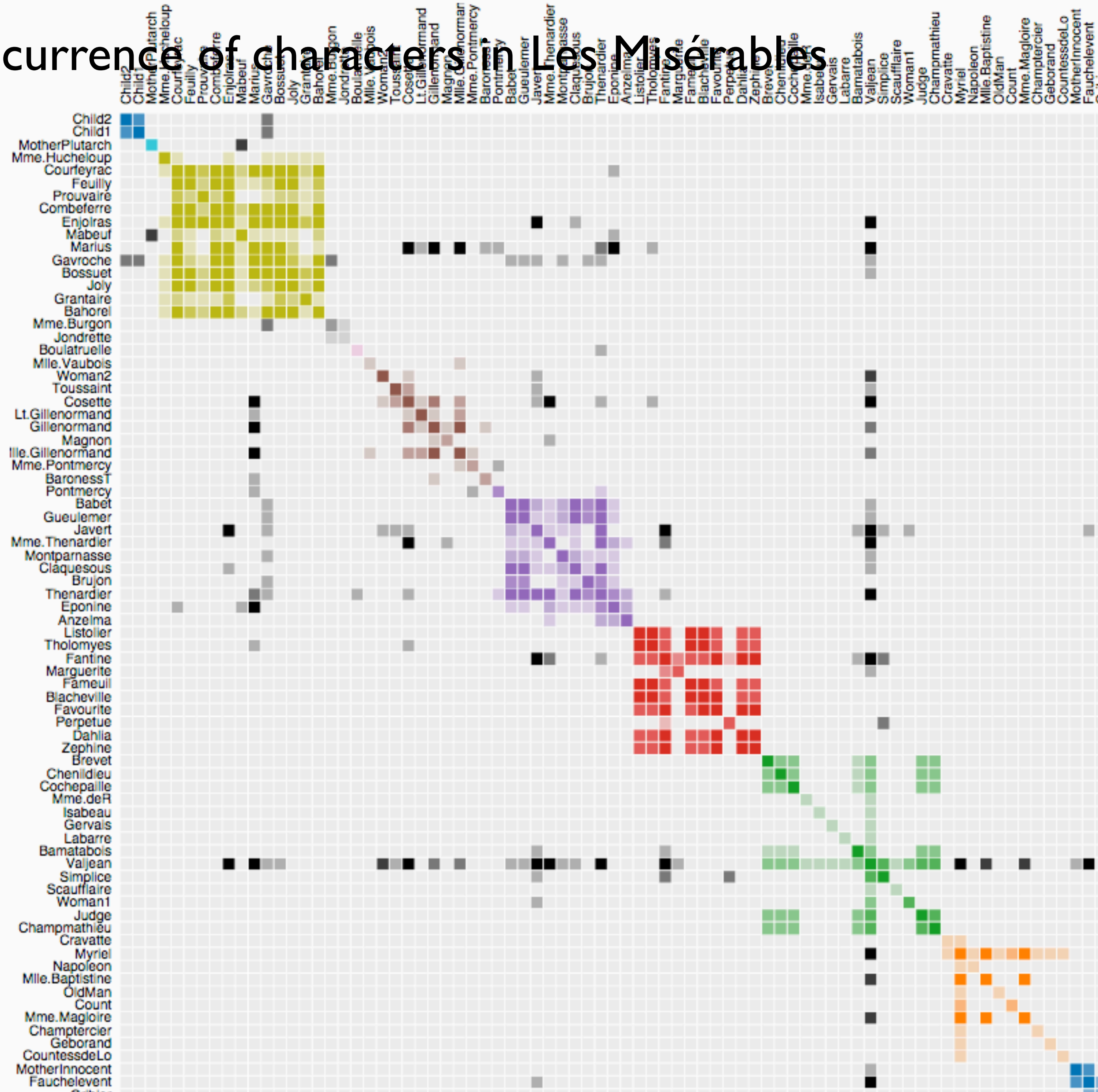
- Why?
  - Edges arrive in no particular order
  - Sometimes, we want to store inverted edges
- Advantages:
  - Supports the ability to perform edge partitioning
- Disadvantages:
  - Takes a lot of space

Co-occurrence of characters in Les Misérables



Source: http://bost.ocks.org/mike/miserables/

Co-occurrence of characters in Les Misérables



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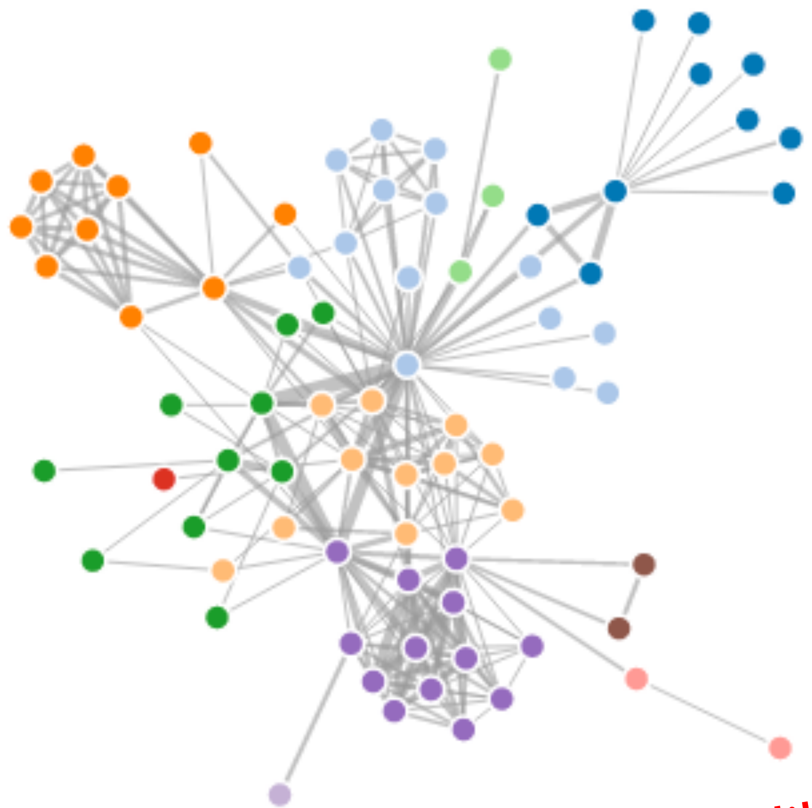


How are visualizations like this generated?
Limitations?

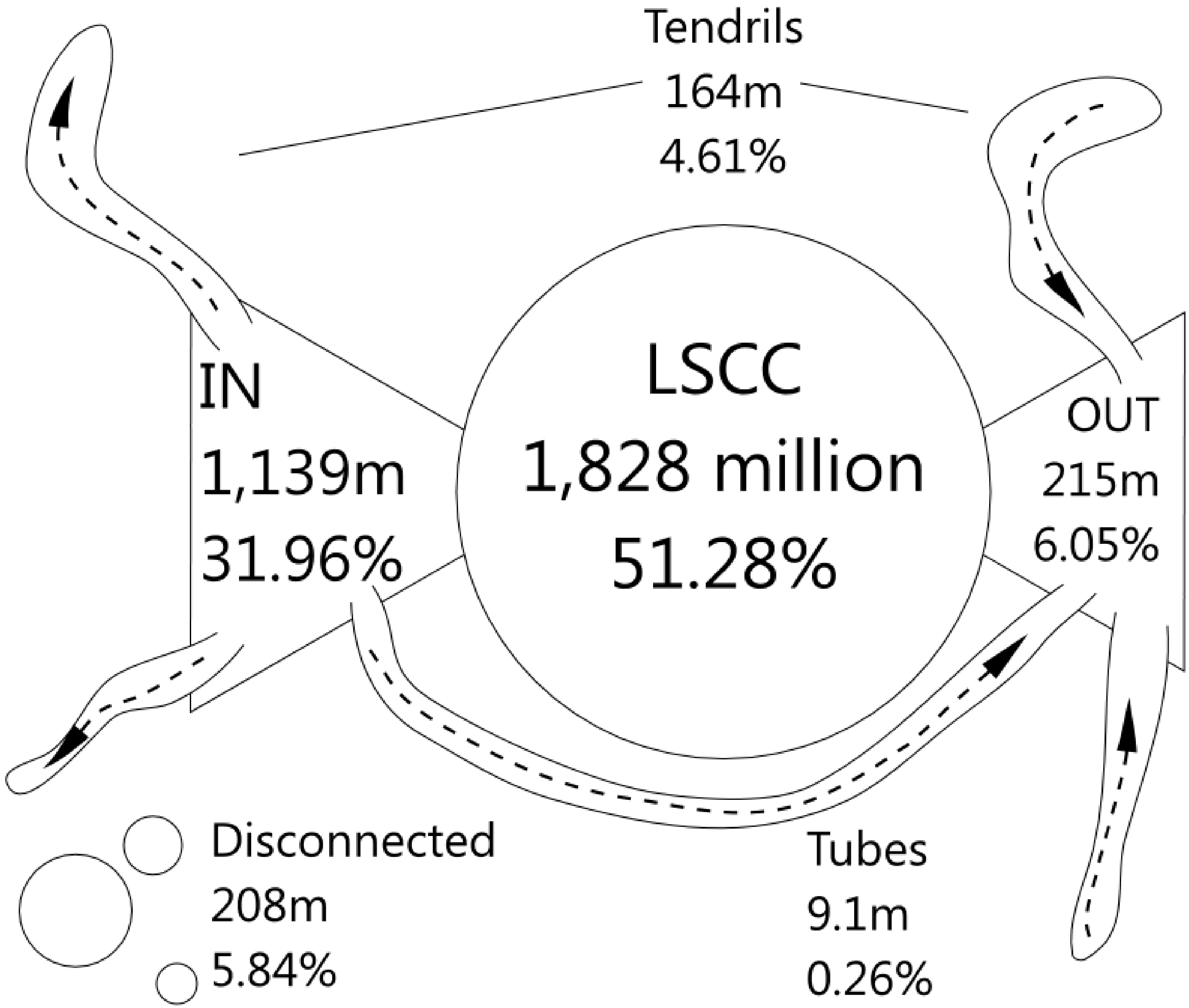
Source: http://bost.ocks.org/mike/miserables/

What does the web look like?

Analysis of a large webgraph from the common crawl: 3.5 billion pages, 129 billion links

Meusel et al. Graph Structure in the Web — Revisited. WWW 2014.

Broder's Bowtie (2000) – revisited



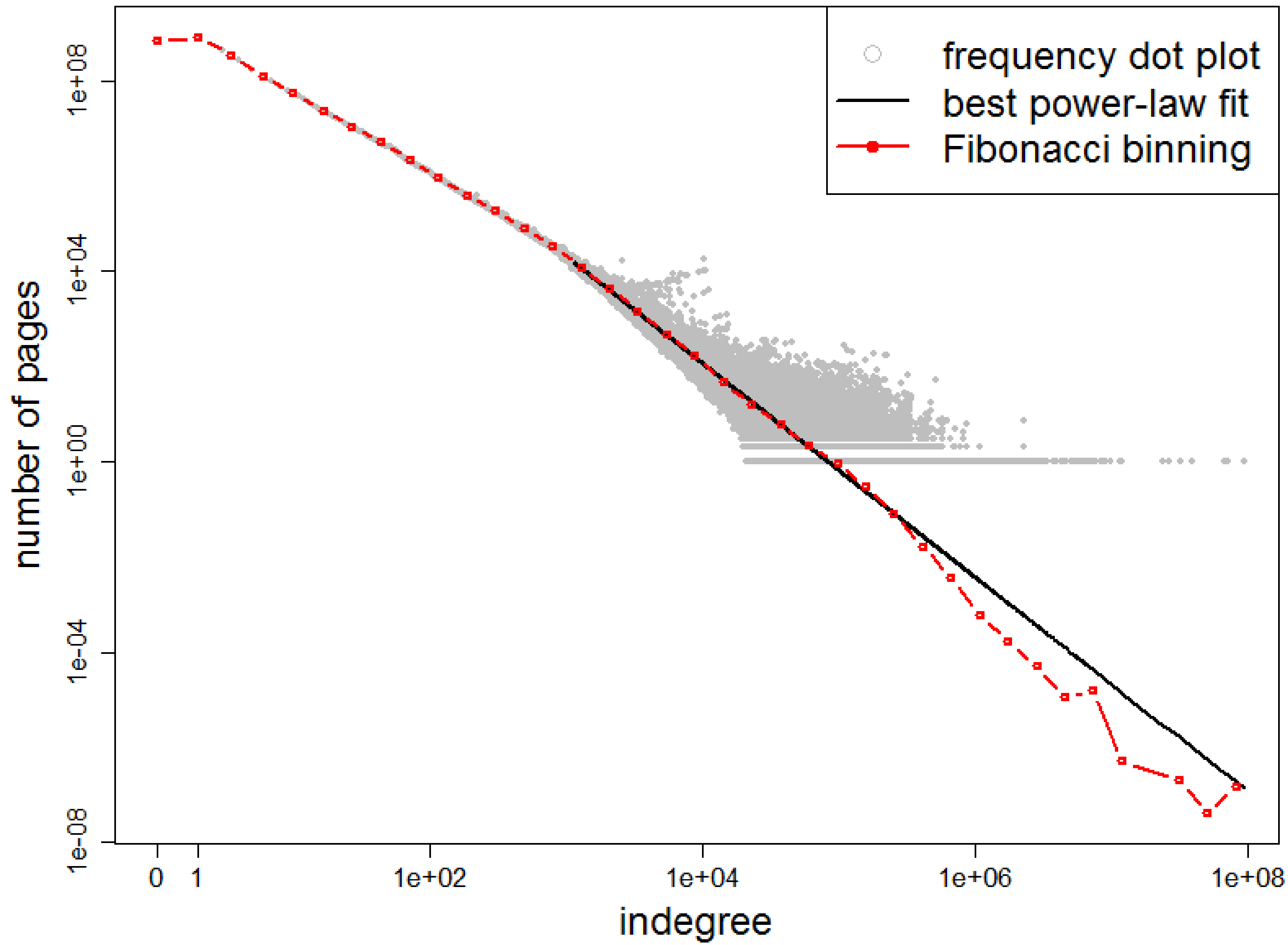
What does the web look like?

Very roughly, a scale-free network

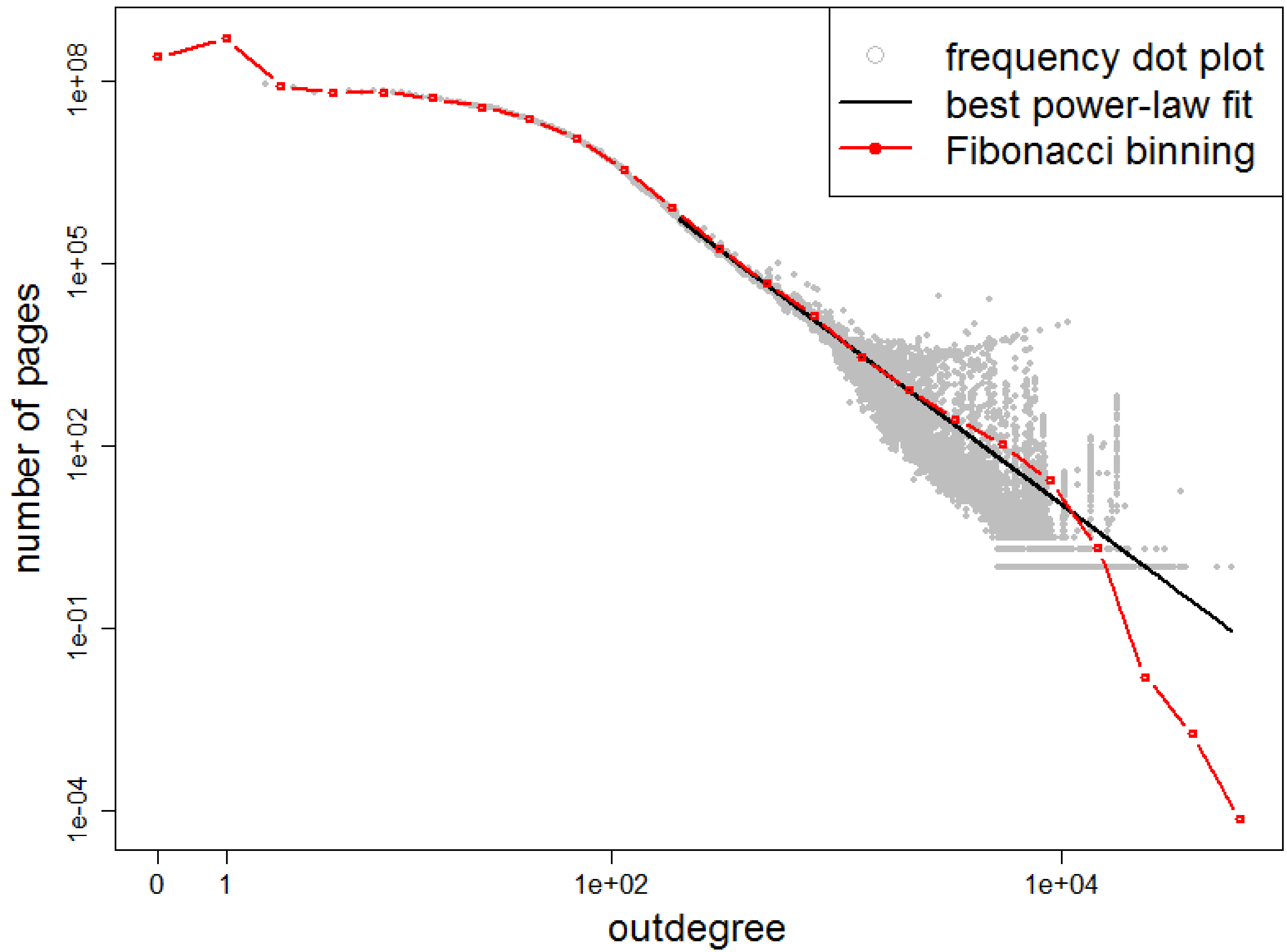
Fraction of k nodes having k connections:

$$P(k) \sim k^{-\gamma}$$

(i.e., distribution follows a power law)



frequency dot plot
best power-law fit
Fibonacci binning
number of pages
1e+08
1e+04
1e+00
1e-04
1e-08
0
1
1e+02
1e+04
1e+06
1e+08
indegree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0
1
1e+02
1e+04
outdegree

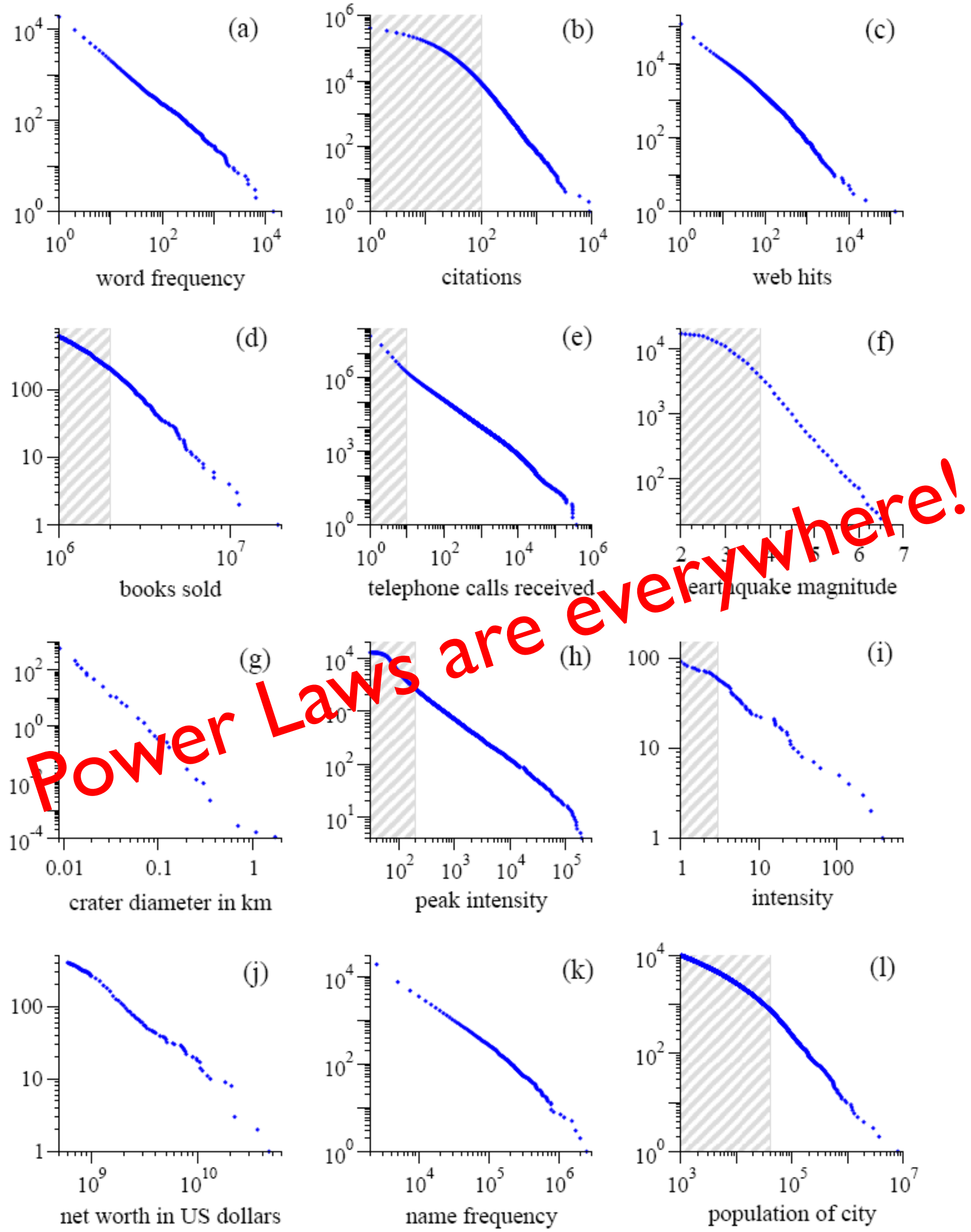


Figure from: Newman, M. E. J. (2005) "Power laws, Pareto distributions and Zipf's law." Contemporary Physics 46:323–351.

What does the web look like?

Very roughly, a scale-free network

Other Examples:

Internet domain routers
Co-author network
Citation network
Movie-Actor network

Why?

(In this installment of “learn fancy terms for simple ideas”)

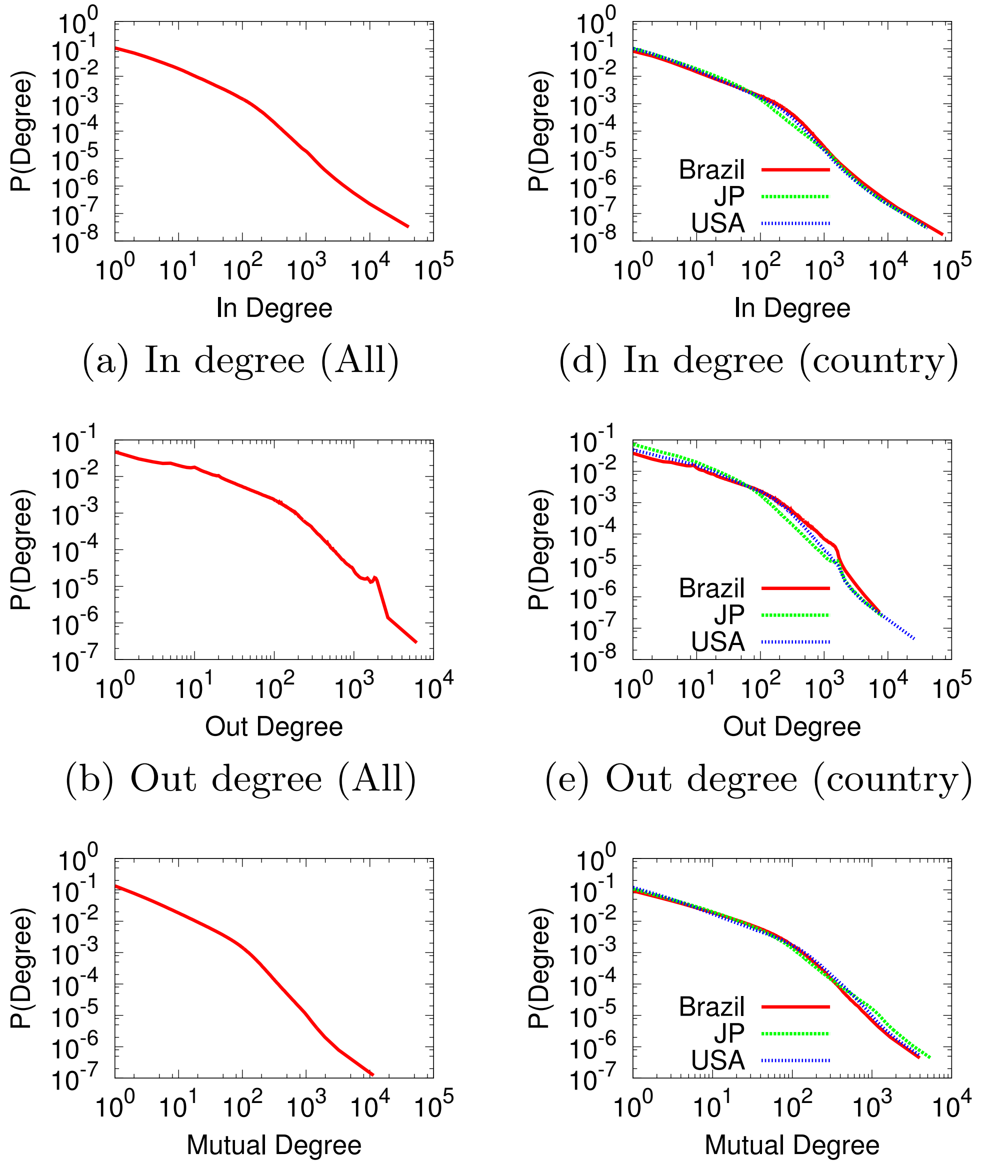
Preferential Attachment

Also:

Matthew Effect

For unto every one that hath shall be given, and he shall have abundance:
but from him that hath not shall be taken even that which he hath.

—Matthew 25:29, King James Version.



(a) In degree (All)
(d) In degree (country)
(b) Out degree (All)
(e) Out degree (country)
(c) Mutual degree (All)
(f) Mutual degree (country)

What about Facebook?

Figure from: Seth A. Myers, Aneesh Sharma, Pankaj Gupta, and Jimmy Lin. Information Network or Social Network? The Structure of the Twitter Follow Graph. WWW 2014.

BTW, how do we compute these graphs?

(Assume graph stored as adjacency lists)

Source: http://www.flickr.com/photos/guvnah/7861418602/

BTW, how do we extract the webgraph?
The webgraph… is big?!

A few tricks:

Integerize vertices (montone minimal perfect hashing)

Sort URLs

Integer compression

webgraph from the common crawl: 3.5 billion pages, 129 billion links

Meusel et al. Graph Structure in the Web — Revisited. WWW 2014.

58 GB!

Graphs and MapReduce (and Spark)

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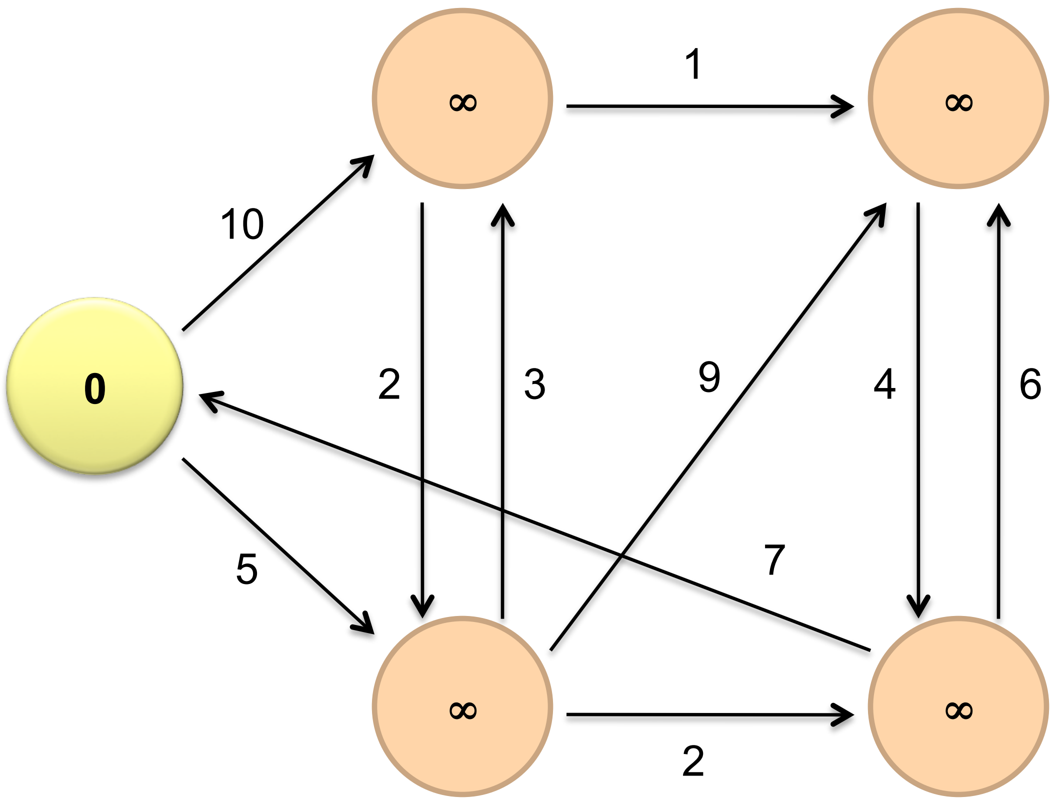
- Key questions:
  - How do you represent graph data in MapReduce (and Spark)?
  - How do you traverse a graph in MapReduce (and Spark)?

Single-Source Shortest Path

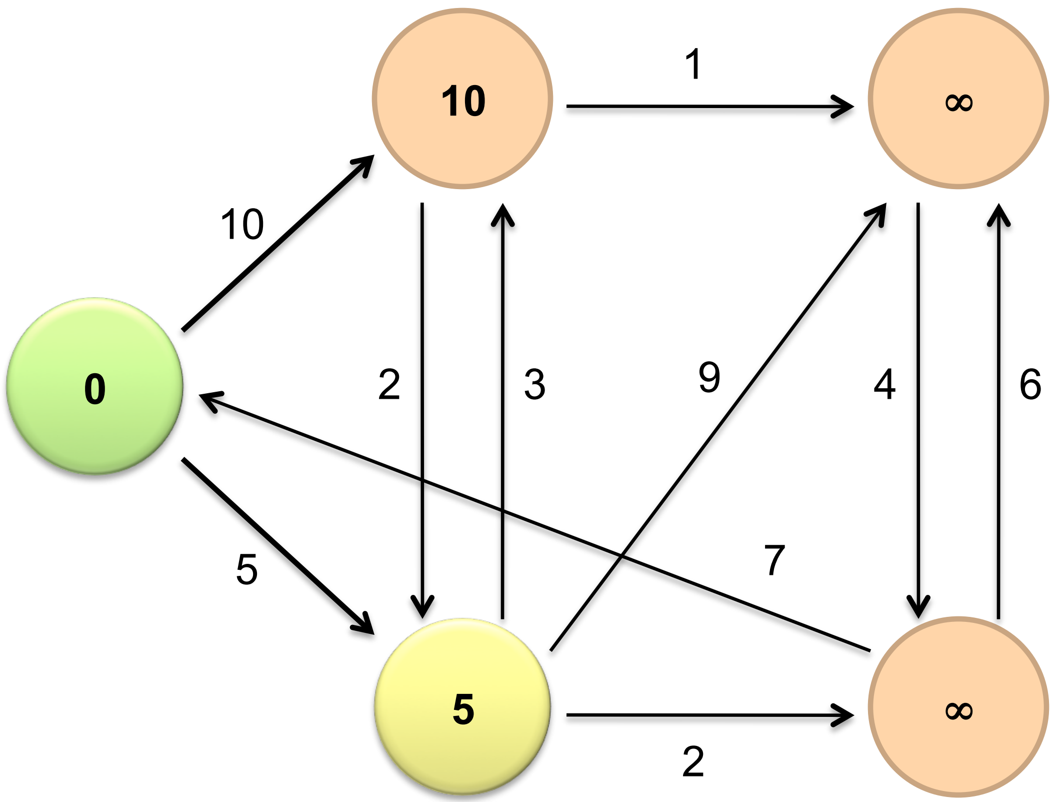
- **Problem:** find shortest path from a source node to one or more target nodes
  - Shortest might also mean lowest weight or cost
- First, a refresher: Dijkstra's Algorithm

Dijkstra's Algorithm Example



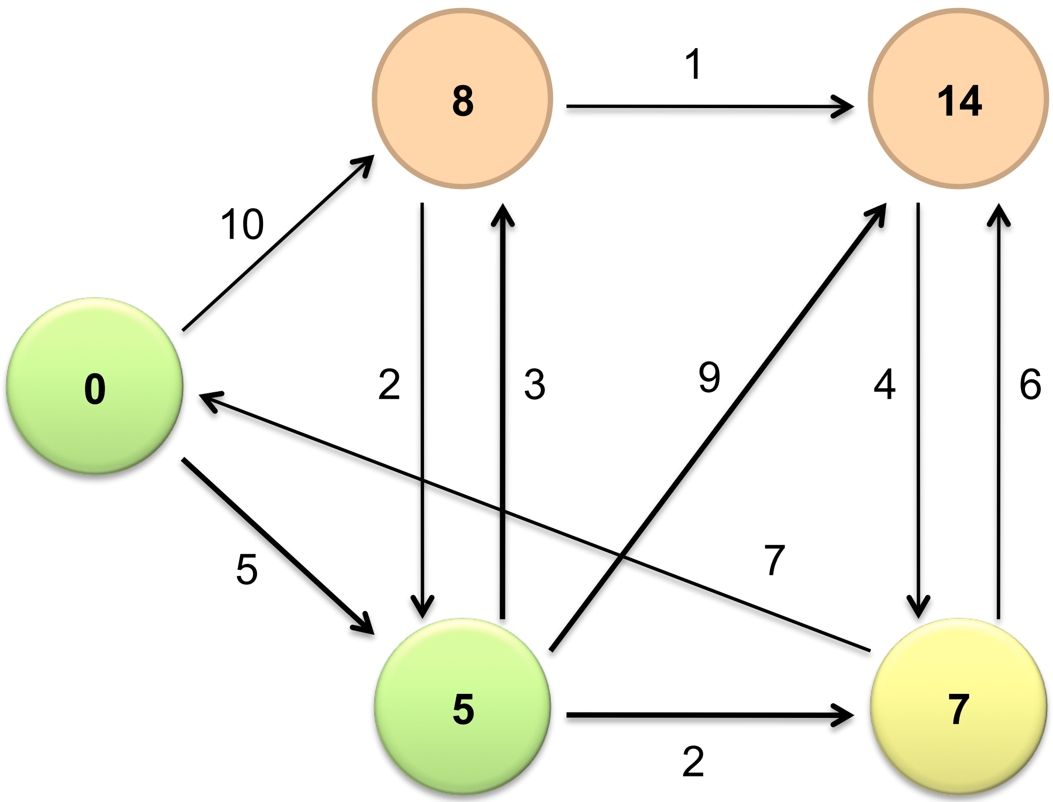
Example from CLR

Dijkstra's Algorithm Example



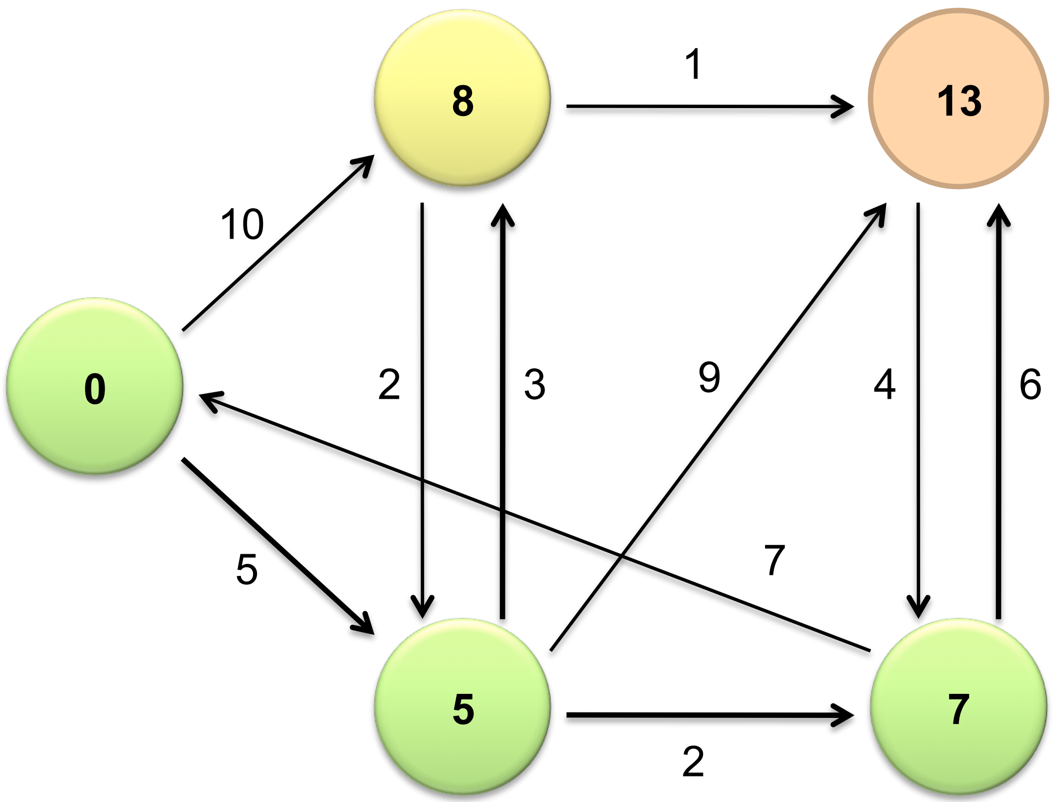
Example from CLR

Dijkstra's Algorithm Example



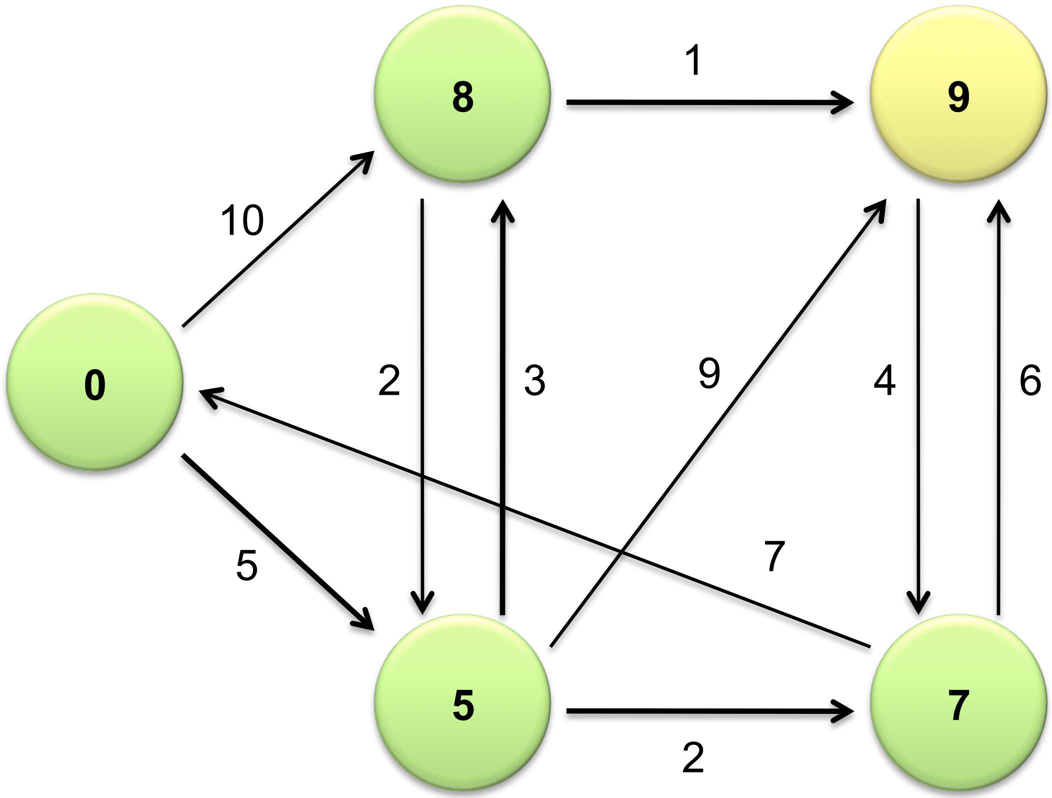
Example from CLR

Dijkstra’s Algorithm Example



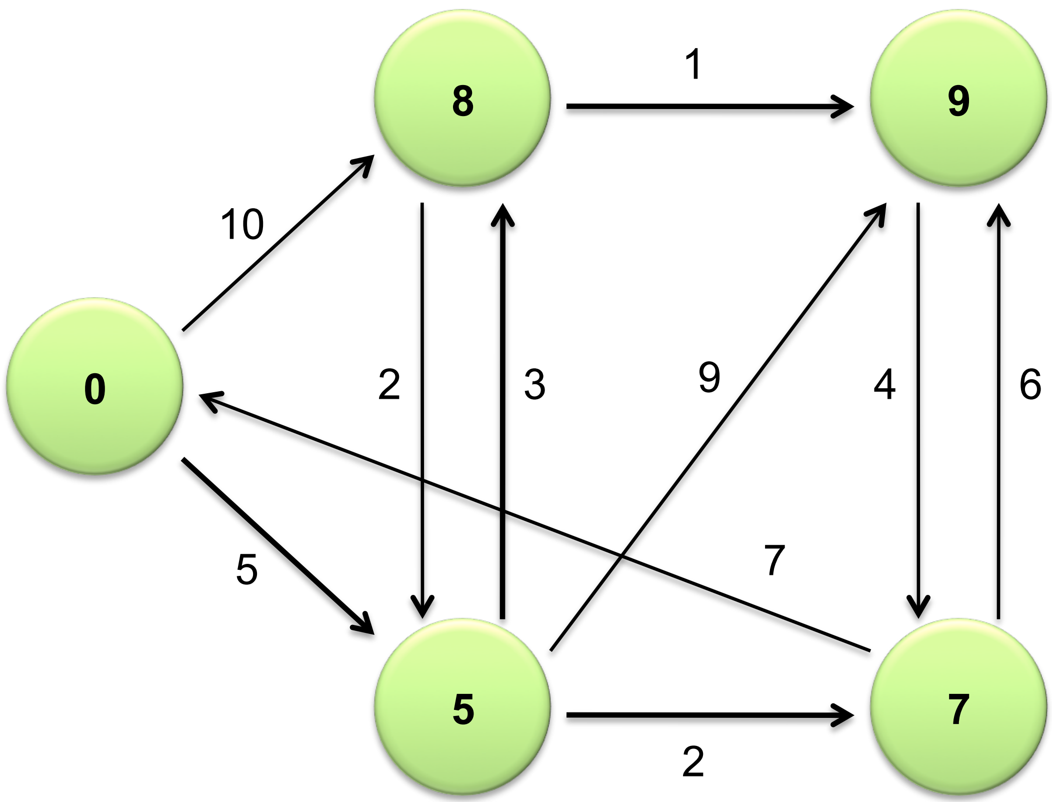
Example from CLR

Dijkstra's Algorithm Example



Example from CLR

Dijkstra's Algorithm Example



Example from CLR

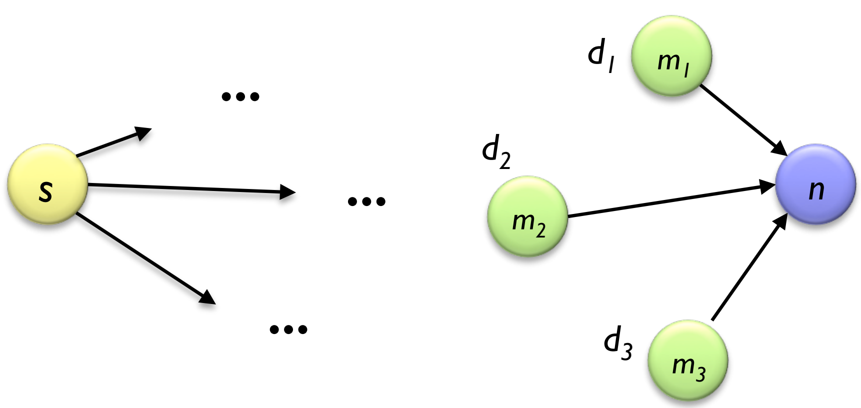
Single-Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
  - Shortest might also mean lowest weight or cost
- Single processor machine: Dijkstra's Algorithm
- MapReduce: parallel breadth-first search (BFS)

Finding the Shortest Path

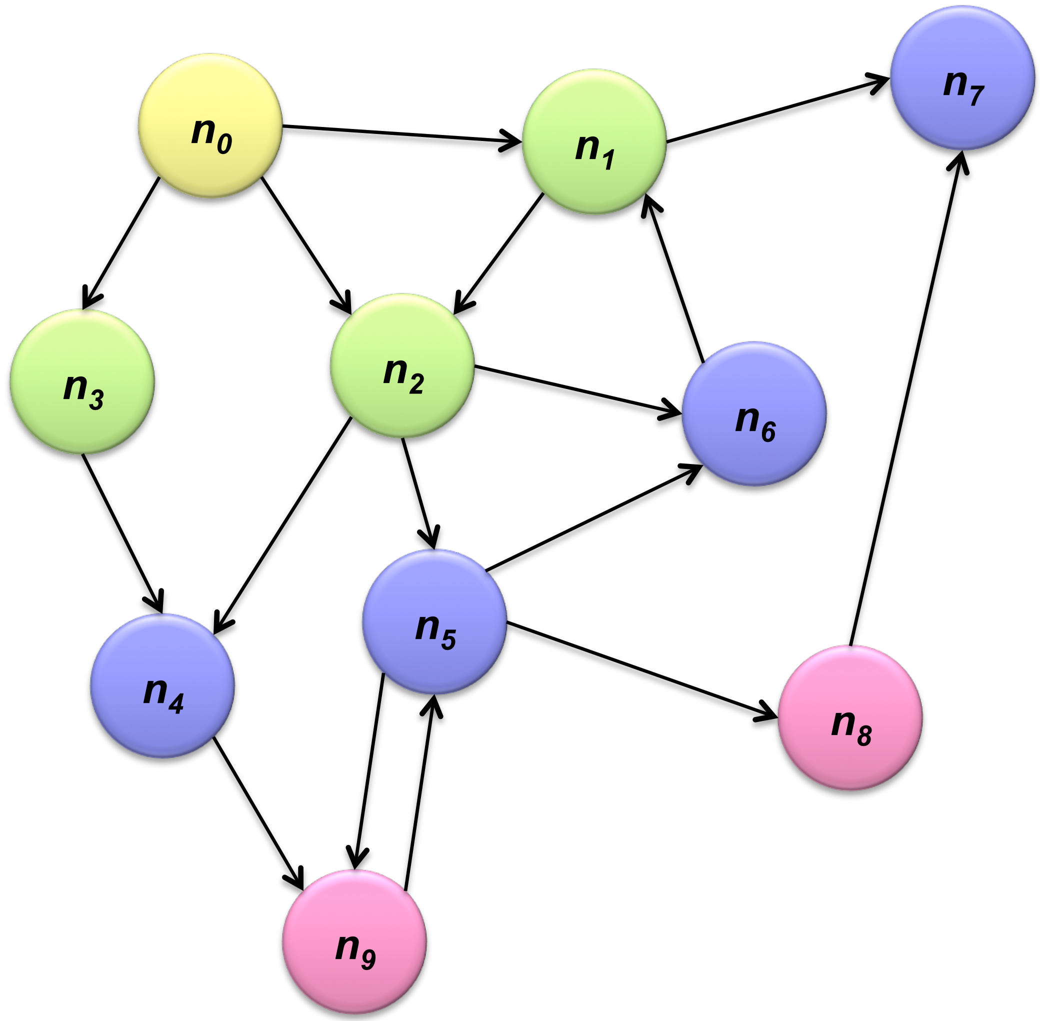
- Consider simple case of equal edge weights
- Solution to the problem can be defined inductively
- Here's the intuition:
  - Define: *b* is reachable from *a* if *b* is on adjacency list of *a*

    DISTANCETO(s) = 0
  - For all nodes p reachable from s,
    DISTANCETO(p) = 1
  - For all nodes n reachable from some other set of nodes M,
    DISTANCETO(n) = 1 + min(DISTANCETO(m), $m \in M$)



Source: Wikipedia (Wave)

Visualizing Parallel BFS



From Intuition to Algorithm

- Data representation:
  - Key: node n
  - Value: d (distance from start), adjacency list (nodes reachable from n)
  - Initialization: for all nodes except for start node, $d = \infty$
- Mapper:
  - $\forall m \in$ adjacency list: emit $(m, d + 1)$
  - Remember to also emit distance to yourself
- Sort/Shuffle
  - Groups distances by reachable nodes
- Reducer:
  - Selects minimum distance path for each reachable node
  - Additional bookkeeping needed to keep track of actual path

Multiple Iterations Needed

- Each MapReduce iteration advances the "frontier" by one hop
  - Subsequent iterations include more and more reachable nodes as frontier expands
  - Multiple iterations are needed to explore entire graph
- Preserving graph structure:
  - Problem: Where did the adjacency list go?
  - Solution: mapper emits (*n*, adjacency list) as well

Ugh! This is ugly!

BFS Pseudo-Code

```
class Mapper
    method Map(nid n, node N)
        d ← N.Distance
        Emit(nid n, N)                              ▷ Pass along graph structure
        for all nodeid m ∈ N.AdjacencyList do
            Emit(nid m, d + 1)                      ▷ Emit distances to reachable nodes

class Reducer
    method Reduce(nid m, [d1, d2, . . .])
        dmin ← ∞
        M ← ∅
        for all d ∈ counts [d1, d2, . . .] do
            if IsNode(d) then
                M ← d                               ▷ Recover graph structure
            else if d < dmin then                   ▷ Look for shorter distance
                dmin ← d
        M.Distance ← dmin                           ▷ Update shortest distance
        Emit(nid m, node M)
```

Stopping Criterion

- How many iterations are needed in parallel BFS (equal edge weight case)?
- Convince yourself: when a node is first "discovered", we've found the shortest path
- Now answer the question...
  - Six degrees of separation?
- Practicalities of implementation in MapReduce

Comparison to Dijkstra

- Dijkstra's algorithm is more efficient
  - At each step, only pursues edges from minimum-cost path inside frontier
- MapReduce explores all paths in parallel
  - Lots of "waste"
  - Useful work is only done at the "frontier"
- Why can't we do better using MapReduce?

Single Source: Weighted Edges

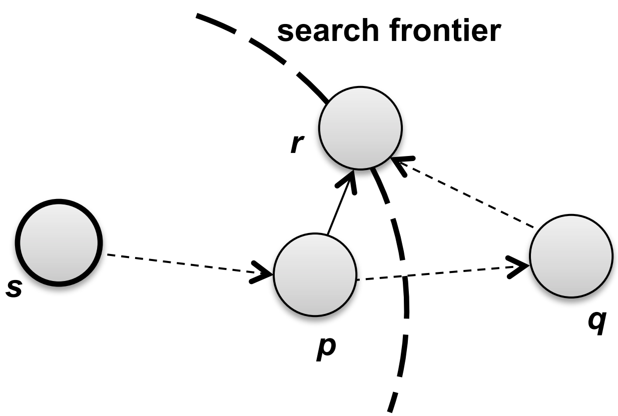
- Now add positive weights to the edges
  - Why can't edge weights be negative?
- Simple change: add weight *w* for each edge in adjacency list
  - In mapper, emit (m, $d + w_p$) instead of (m, $d + 1$) for each node m
- That's it?

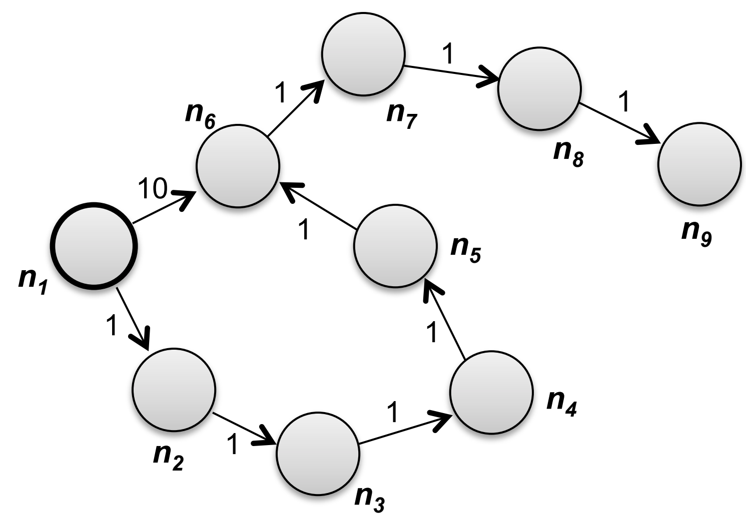
Stopping Criterion

- How many iterations are needed in parallel BFS (positive edge weight case)?
- Convince yourself: when a node is first "discovered", we've found the shortest path

Not true!

Additional Complexities





Stopping Criterion

- How many iterations are needed in parallel BFS (positive edge weight case)?
- Practicalities of implementation in MapReduce

Application: Social Search

Source: Wikipedia (Crowd)

Social Search

- When searching, how to rank friends named “John”?
  - Assume undirected graphs
  - Rank matches by distance to user
- Naïve implementations:
  - Precompute all-pairs distances
  - Compute distances at query time
- Can we do better?

All-Pairs?

- Floyd-Warshall Algorithm: difficult to MapReduce-ify…
- Multiple-source shortest paths in MapReduce: run multiple parallel BFS *simultaneously*
  - Assume source nodes $\{s_0, s_1, \ldots s_n\}$
  - Instead of emitting a single distance, emit an array of distances, with respect to each source
  - Reducer selects minimum for each element in array
- Does this scale?

Landmark Approach (aka sketches)

- Select n seeds $\{s_0, s_1, \ldots s_n\}$
- Compute distances from seeds to every node:

Nodes

A = [2, 1, 1]
B = [1, 1, 2]
C = [4, 3, 1]
D = [1, 2, 4]

Distances to seeds

  - What can we conclude about distances?
  - Insight: landmarks bound the maximum path length
- Lots of details:
  - How to more tightly bound distances
  - How to select landmarks (random isn't the best…)
- Use multi-source parallel BFS implementation in MapReduce!

Source: Wikipedia (Japanese rock garden)