Big Data Infrastructure

Session 5: MapReduce – Graphs

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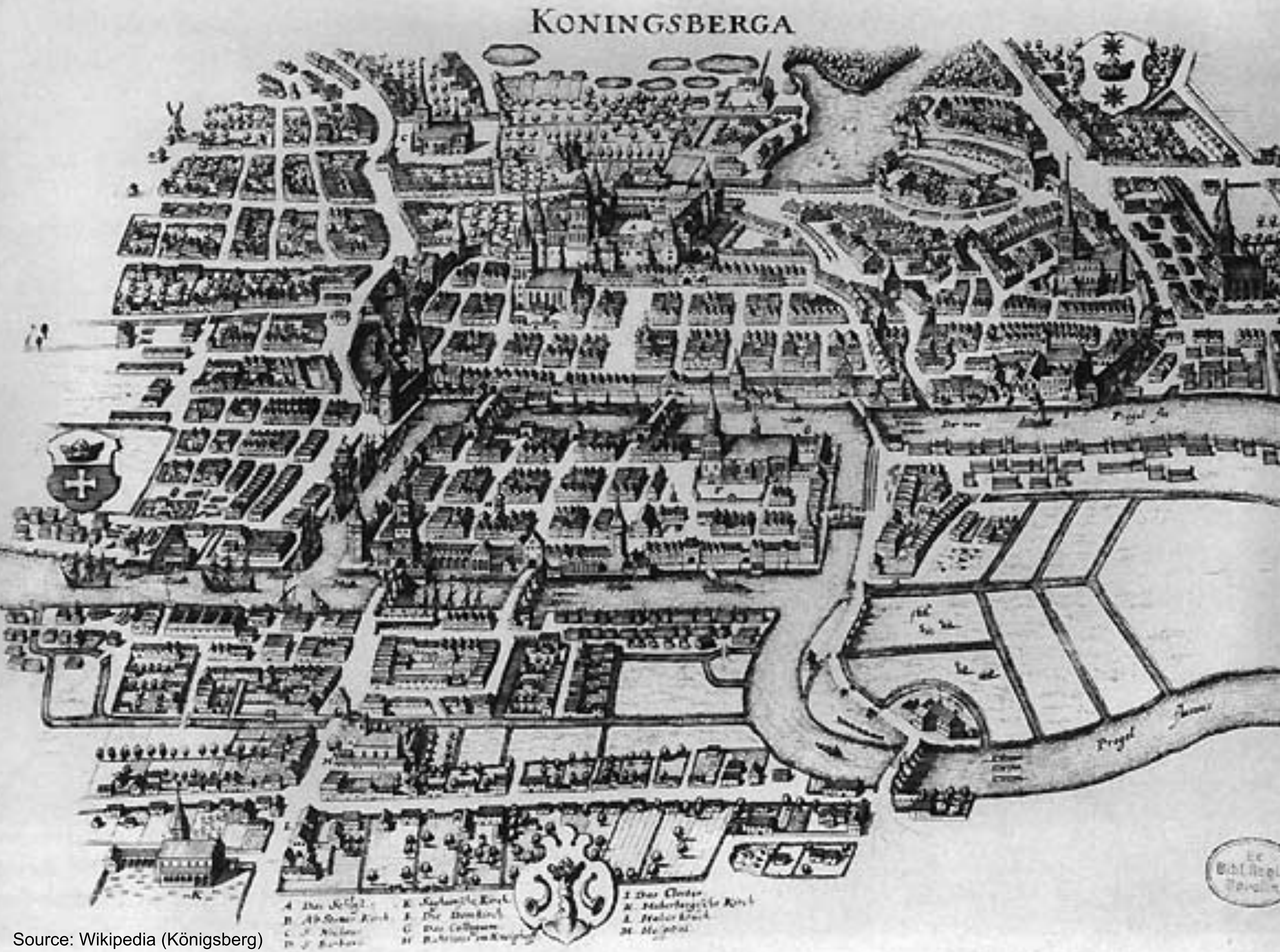
Today's Agenda

- Graph problems and representations
- Parallel breadth-first search
- PageRank
- Beyond PageRank and other graph algorithms
- Optimizing graph algorithms

What's a graph?

- G = (V,E), where
  - V represents the set of vertices (nodes)
  - E represents the set of edges (links)
  - Both vertices and edges may contain additional information
- Different types of graphs:
  - Directed vs. undirected edges
  - Presence or absence of cycles
- Graphs are everywhere:
  - Hyperlink structure of the web
  - Physical structure of computers on the Internet
  - Interstate highway system
  - Social networks



Source: Wikipedia (Königsberg)

Source: Wikipedia (Kaliningrad)

Some Graph Problems

- Finding shortest paths
  - Routing Internet traffic and UPS trucks
- Finding minimum spanning trees
  - Telco laying down fiber
- Finding Max Flow
  - Airline scheduling
- Identify “special” nodes and communities
  - Breaking up terrorist cells, spread of avian flu
- Bipartite matching
  - Monster.com, Match.com
- And of course... PageRank

Graphs and MapReduce

- A large class of graph algorithms involve:
  - Performing computations at each node: based on node features, edge features, and local link structure
  - Propagating computations: “traversing” the graph
- Key questions:
  - How do you represent graph data in MapReduce?
  - How do you traverse a graph in MapReduce?

Representing Graphs

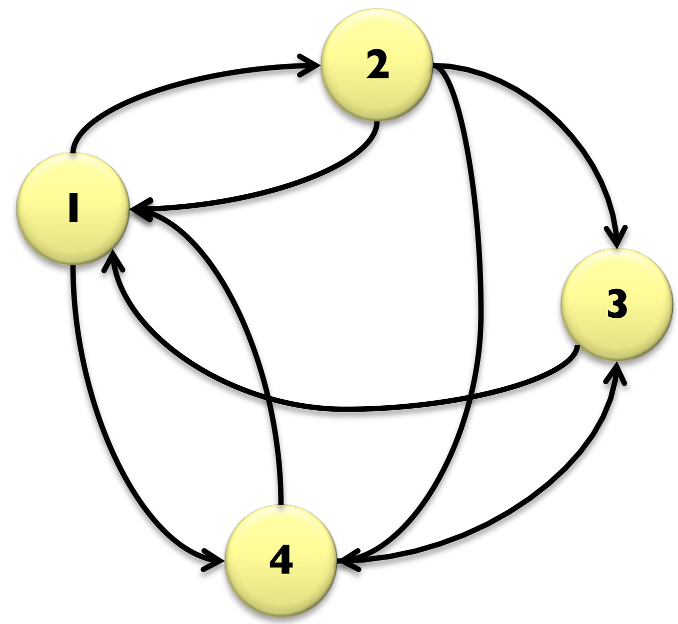
- G = (V, E)
- Two common representations
  - Adjacency matrix
  - Adjacency list

Adjacency Matrices

Represent a graph as an n x n square matrix M

- $n = |V|$
- $M_{ij} = 1$ means a link from node i to j

	1	2	3	4
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0



Adjacency Matrices: Critique

- Advantages:
  - Amenable to mathematical manipulation
  - Iteration over rows and columns corresponds to computations on outlinks and inlinks
- Disadvantages:
  - Lots of zeros for sparse matrices
  - Lots of wasted space

Adjacency Lists

Take adjacency matrices… and throw away all the zeros

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

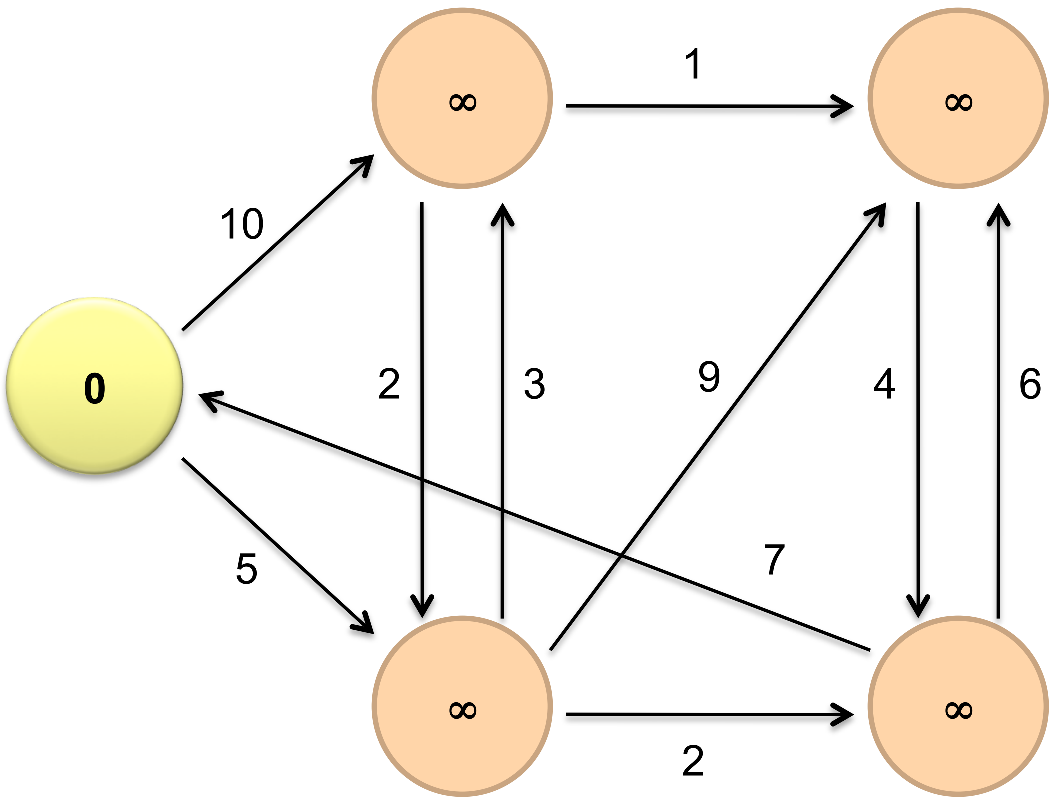
Adjacency Lists: Critique

- Advantages:
  - Much more compact representation
  - Easy to compute over outlinks
- Disadvantages:
  - Much more difficult to compute over inlinks

Single-Source Shortest Path

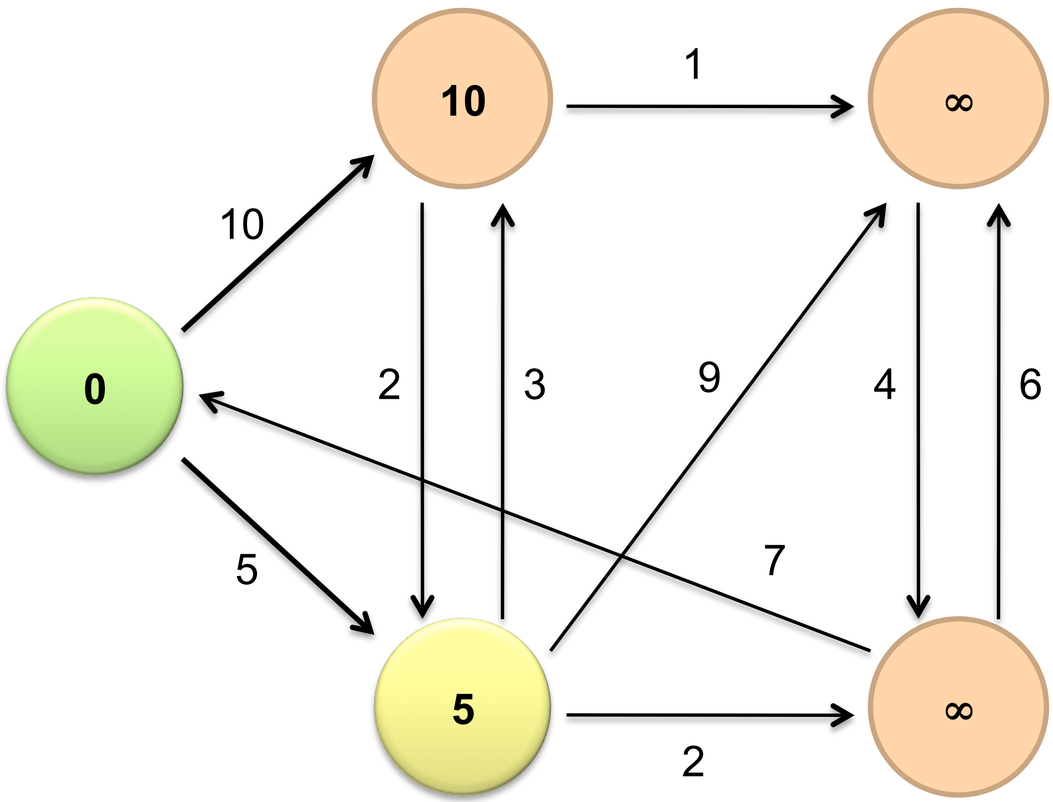
- **Problem:** find shortest path from a source node to one or more target nodes
  - Shortest might also mean lowest weight or cost
- First, a refresher: Dijkstra's Algorithm

Dijkstra’s Algorithm Example



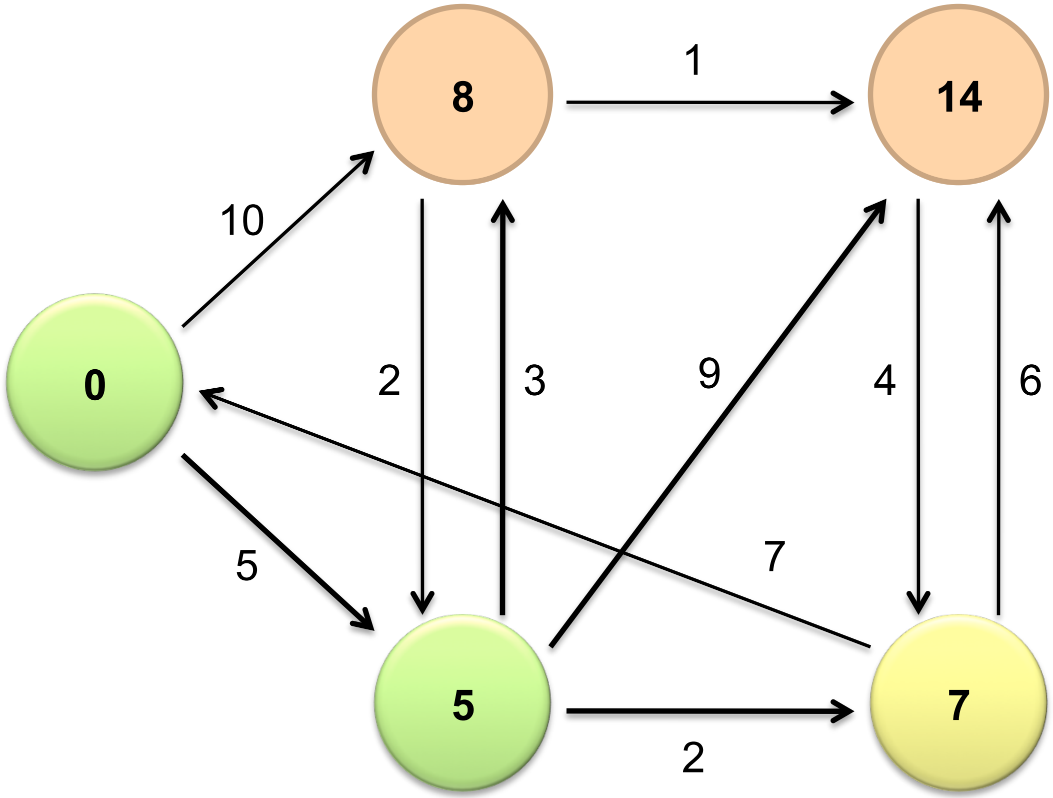
Example from CLR

Dijkstra’s Algorithm Example



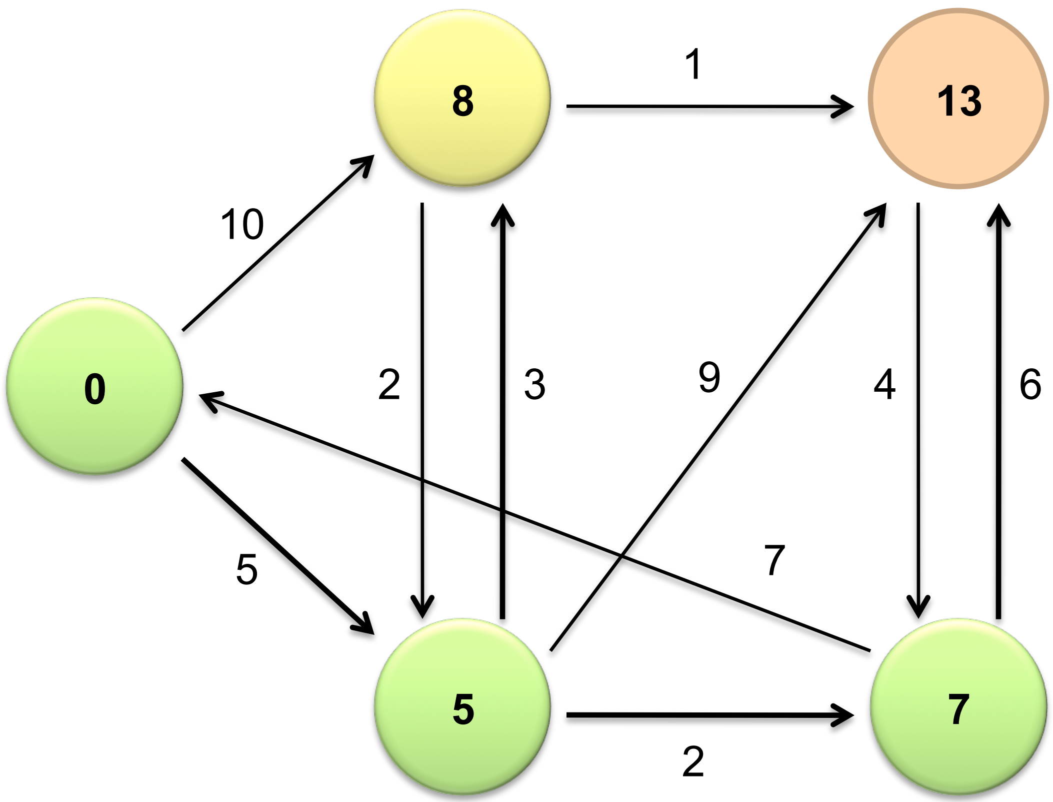
Example from CLR

Dijkstra's Algorithm Example



Example from CLR

Dijkstra's Algorithm Example



Example from CLR

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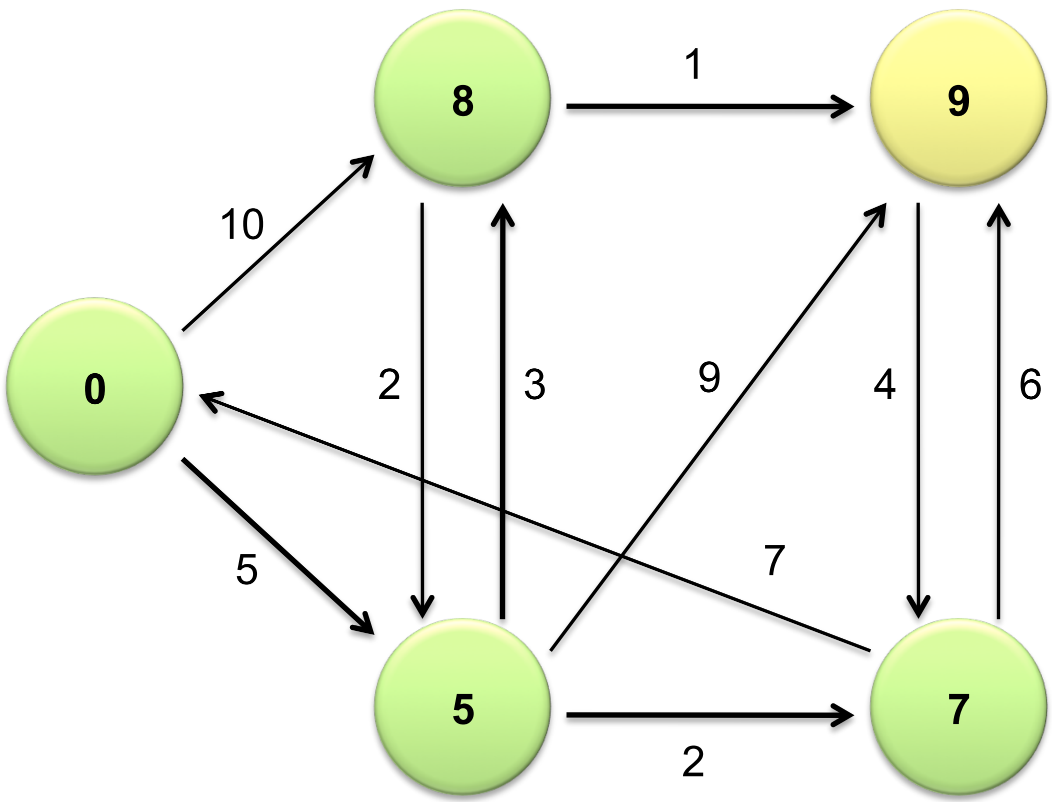
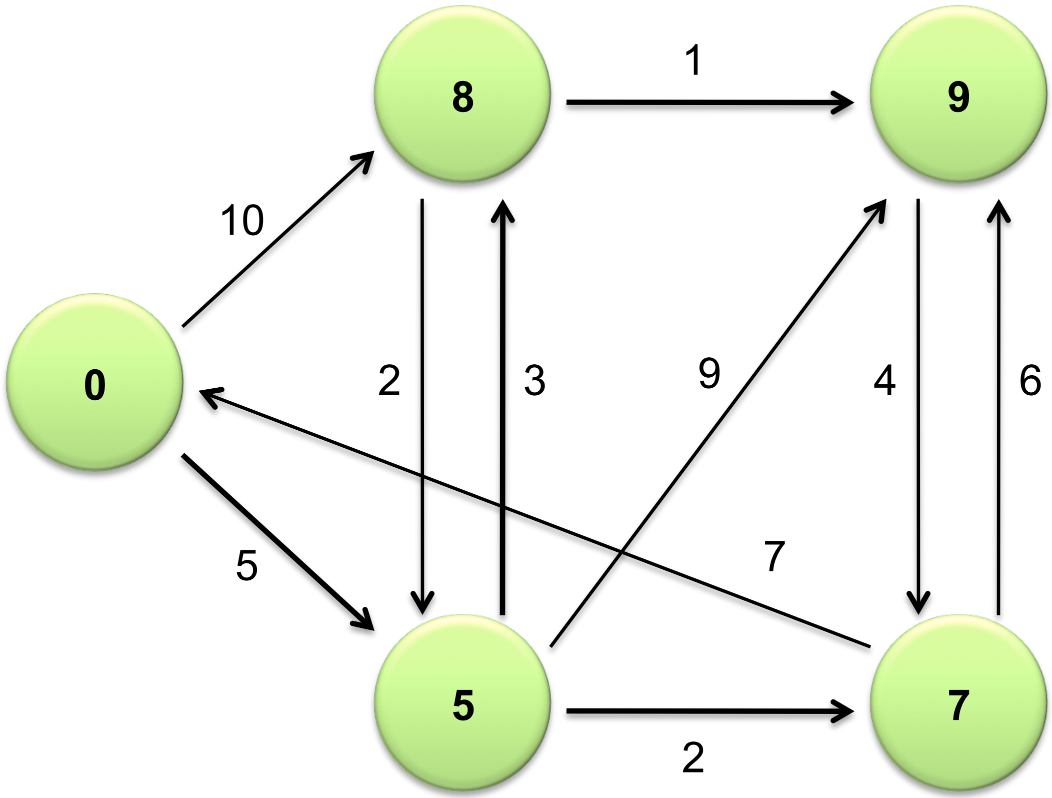


Example from CLR

Dijkstra's Algorithm Example



Example from CLR

Single-Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
  - Shortest might also mean lowest weight or cost
- Single processor machine: Dijkstra's Algorithm
- MapReduce: parallel breadth-first search (BFS)

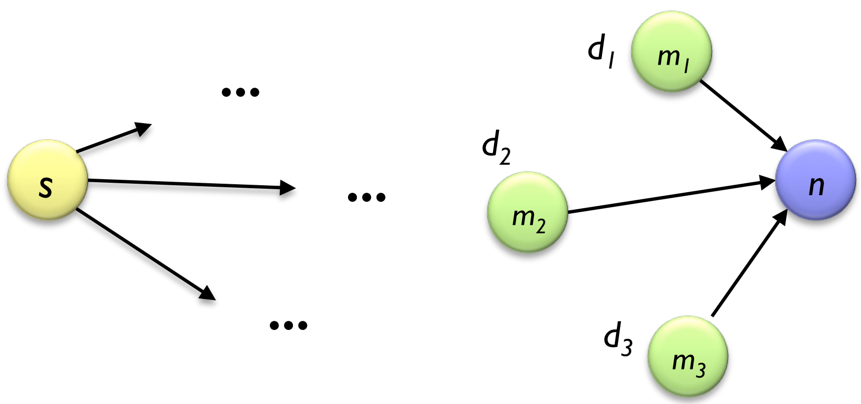
Finding the Shortest Path

- Consider simple case of equal edge weights
- Solution to the problem can be defined inductively
- Here's the intuition:
  - Define: *b* is reachable from *a* if *b* is on adjacency list of *a*

    DistanceTo(s) = 0
  - For all nodes p reachable from s,

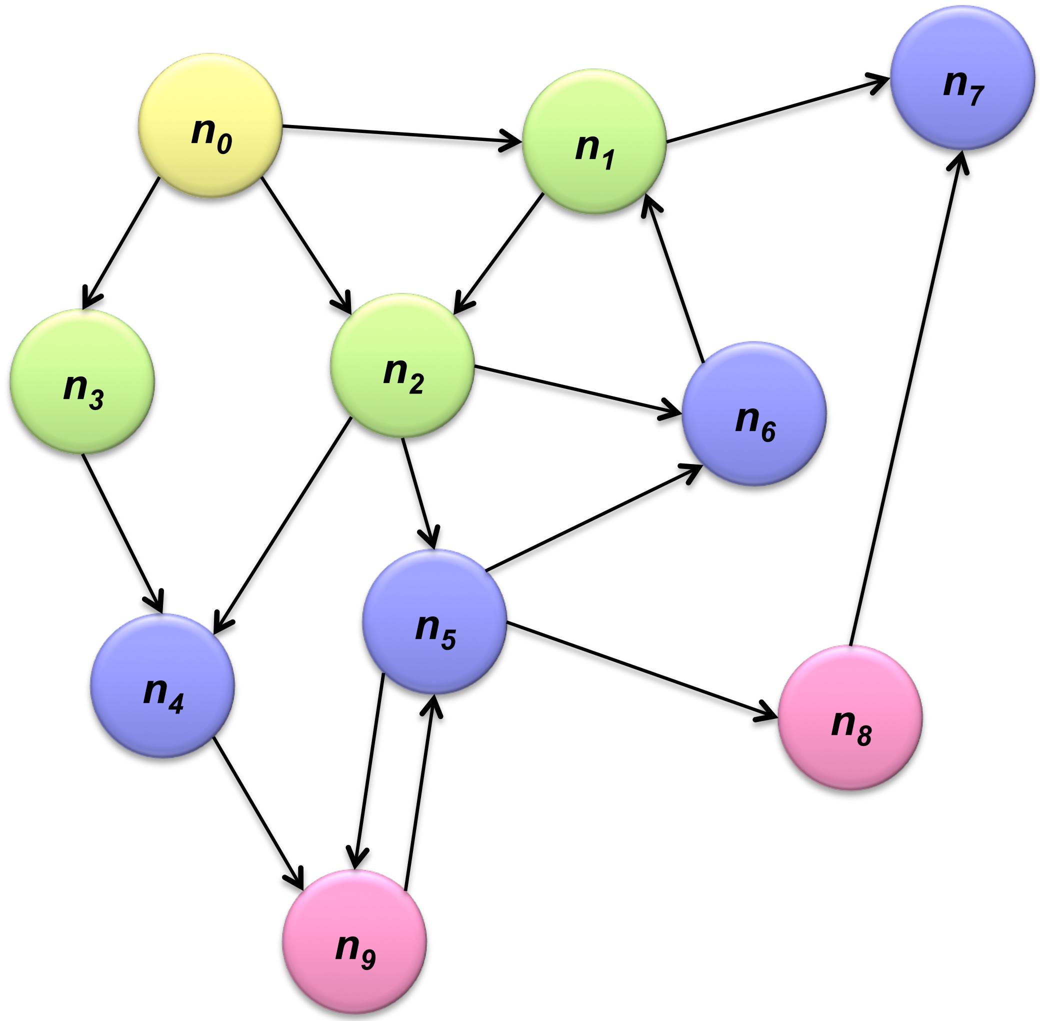
    DistanceTo(p) = 1
  - For all nodes n reachable from some other set of nodes M,

    DistanceTo(n) = 1 + min(DistanceTo(m), $m \in M$)



Source: Wikipedia (Wave)

Visualizing Parallel BFS



From Intuition to Algorithm

- Data representation:
  - Key: node n
  - Value: d (distance from start), adjacency list (nodes reachable from n)
  - Initialization: for all nodes except for start node, $d = \infty$
- Mapper:
  - $\forall m \in$ adjacency list: emit $(m, d + 1)$
  - Remember to also emit distance to yourself
- Sort/Shuffle
  - Groups distances by reachable nodes
- Reducer:
  - Selects minimum distance path for each reachable node
  - Additional bookkeeping needed to keep track of actual path

Multiple Iterations Needed

- Each MapReduce iteration advances the “frontier” by one hop
  - Subsequent iterations include more and more reachable nodes as frontier expands
  - Multiple iterations are needed to explore entire graph
- Preserving graph structure:
  - Problem: Where did the adjacency list go?
  - Solution: mapper emits (*n*, adjacency list) as well

BFS Pseudo-Code

```
class Mapper
    method Map(nid n, node N)
        d ← N.Distance
        Emit(nid n, N)                          ▷ Pass along graph structure
        for all nodeid m ∈ N.AdjacencyList do
            Emit(nid m, d + 1)                  ▷ Emit distances to reachable nodes

class Reducer
    method Reduce(nid m, [d1, d2, ...])
        d_min ← ∞
        M ← ∅
        for all d ∈ counts [d1, d2, ...] do
            if IsNode(d) then
                M ← d                           ▷ Recover graph structure
            else if d < d_min then              ▷ Look for shorter distance
                d_min ← d
        M.Distance ← d_min                      ▷ Update shortest distance
        Emit(nid m, node M)
```

Stopping Criterion

- How many iterations are needed in parallel BFS (equal edge weight case)?
- Convince yourself: when a node is first "discovered", we've found the shortest path
- Now answer the question...
  - Six degrees of separation?
- Practicalities of implementation in MapReduce

Comparison to Dijkstra

- Dijkstra's algorithm is more efficient
  - At each step, only pursues edges from minimum-cost path inside frontier
- MapReduce explores all paths in parallel
  - Lots of "waste"
  - Useful work is only done at the "frontier"
- Why can't we do better using MapReduce?

Single Source: Weighted Edges

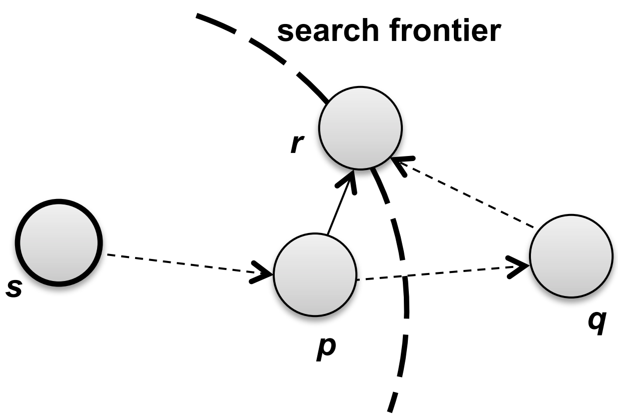
- Now add positive weights to the edges
  - Why can't edge weights be negative?
- Simple change: add weight *w* for each edge in adjacency list
  - In mapper, emit $(m, d + w_p)$ instead of $(m, d + 1)$ for each node m
- That's it?

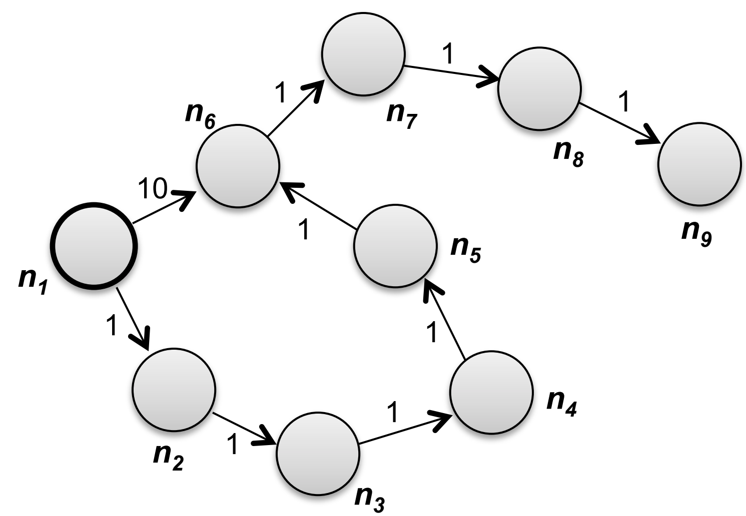
Stopping Criterion

- How many iterations are needed in parallel BFS (positive edge weight case)?
- Convince yourself: when a node is first "discovered", we've found the shortest path

Not true!

Additional Complexities





Stopping Criterion

- How many iterations are needed in parallel BFS (positive edge weight case)?
- Practicalities of implementation in MapReduce

Application: Social Search

Source: Wikipedia (Crowd)

Social Search

- When searching, how to rank friends named "John"?
  - Assume undirected graphs
  - Rank matches by distance to user
- Naïve implementations:
  - Precompute all-pairs distances
  - Compute distances at query time
- Can we do better?

All-Pairs?

- Floyd-Warshall Algorithm: difficult to MapReduce-ify…
- Multiple-source shortest paths in MapReduce: run multiple parallel BFS *simultaneously*
  - Assume source nodes $\{s_0, s_1, \ldots s_n\}$
  - Instead of emitting a single distance, emit an array of distances, with respect to each source
  - Reducer selects minimum for each element in array
- Does this scale?

Landmark Approach (aka sketches)

- Select n seeds $\{s_0, s_1, \ldots s_n\}$
- Compute distances from seeds to every node:

  A = [2, 1, 1]
  B = [1, 1, 2]
  C = [4, 3, 1]
  D = [1, 2, 4]

  - What can we conclude about distances?
  - Insight: landmarks bound the maximum path length
- Lots of details:
  - How to more tightly bound distances
  - How to select landmarks (random isn't the best…)
- Use multi-source parallel BFS implementation in MapReduce!

Source: Wikipedia (Wave)

Graphs and MapReduce

- A large class of graph algorithms involve:
  - Performing computations at each node: based on node features, edge features, and local link structure
  - Propagating computations: "traversing" the graph
- Generic recipe:
  - Represent graphs as adjacency lists
  - Perform local computations in mapper
  - Pass along partial results via outlinks, keyed by destination node
  - Perform aggregation in reducer on inlinks to a node
  - Iterate until convergence: controlled by external "driver"
  - Don't forget to pass the graph structure between iterations

Random Walks Over the Web

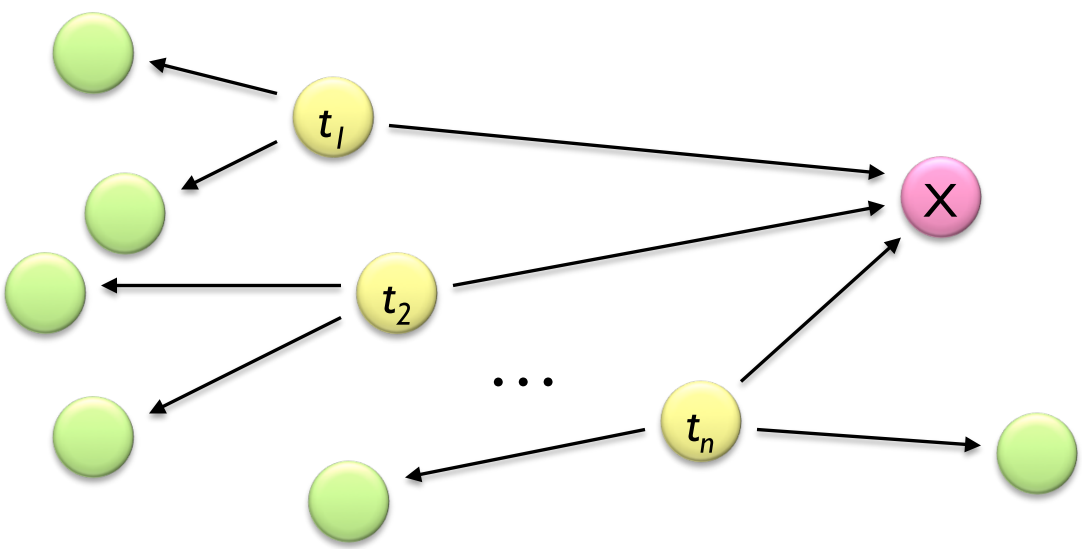
- Random surfer model:
  - User starts at a random Web page
  - User randomly clicks on links, surfing from page to page
- PageRank
  - Characterizes the amount of time spent on any given page
  - Mathematically, a probability distribution over pages
- PageRank captures notions of page importance
  - Correspondence to human intuition?
  - One of thousands of features used in web search (query-independent)

PageRank: Defined

Given page x with inlinks $t_1 \ldots t_n$, where

- $C(t)$ is the out-degree of t
- α is probability of random jump
- N is the total number of nodes in the graph

$$PR(x) = \alpha \left(\frac{1}{N} \right) + (1 - \alpha) \sum_{i=1}^{n} \frac{PR(t_i)}{C(t_i)}$$



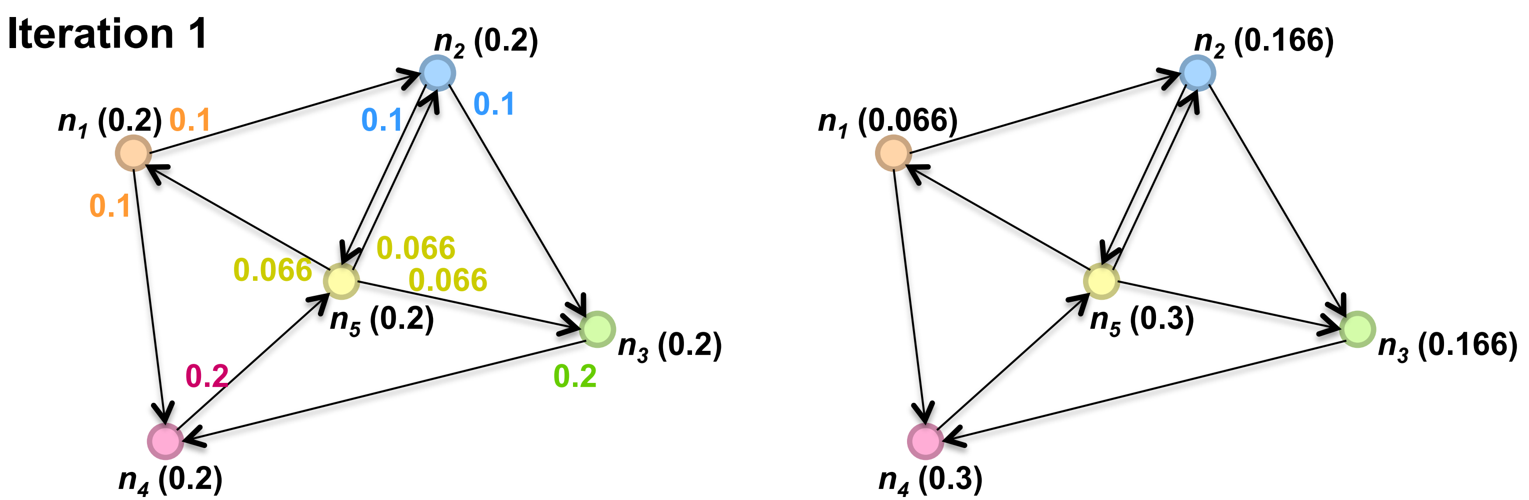
Computing PageRank

- Properties of PageRank
  - Can be computed iteratively
  - Effects at each iteration are local
- Sketch of algorithm:
  - Start with seed PR_i values
  - Each page distributes PR_i "credit" to all pages it links to
  - Each target page adds up "credit" from multiple in-bound links to compute PR_{i+1}
  - Iterate until values converge

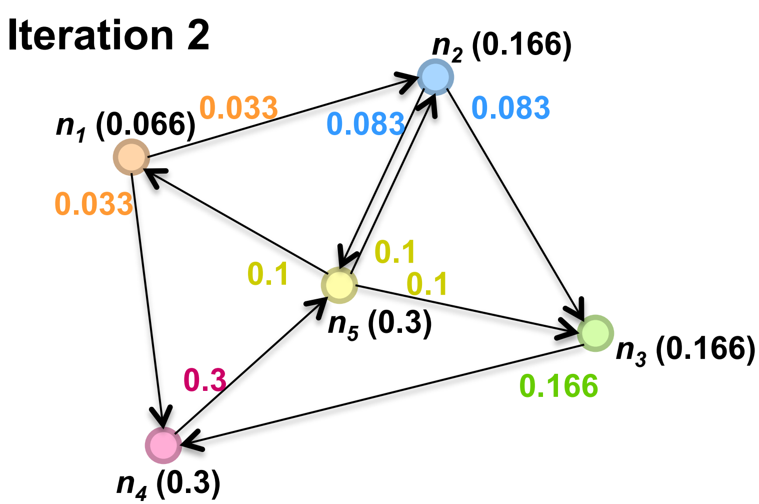
Simplified PageRank

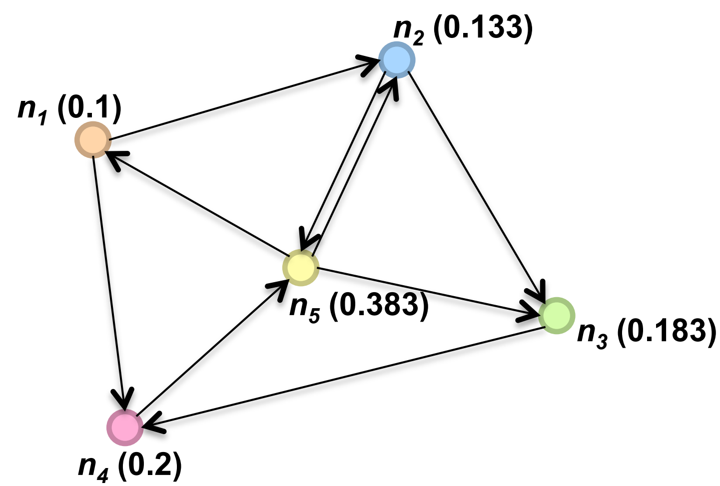
- First, tackle the simple case:
  - No random jump factor
  - No dangling nodes
- Then, factor in these complexities…
  - Why do we need the random jump?
  - Where do dangling nodes come from?

Sample PageRank Iteration (1)

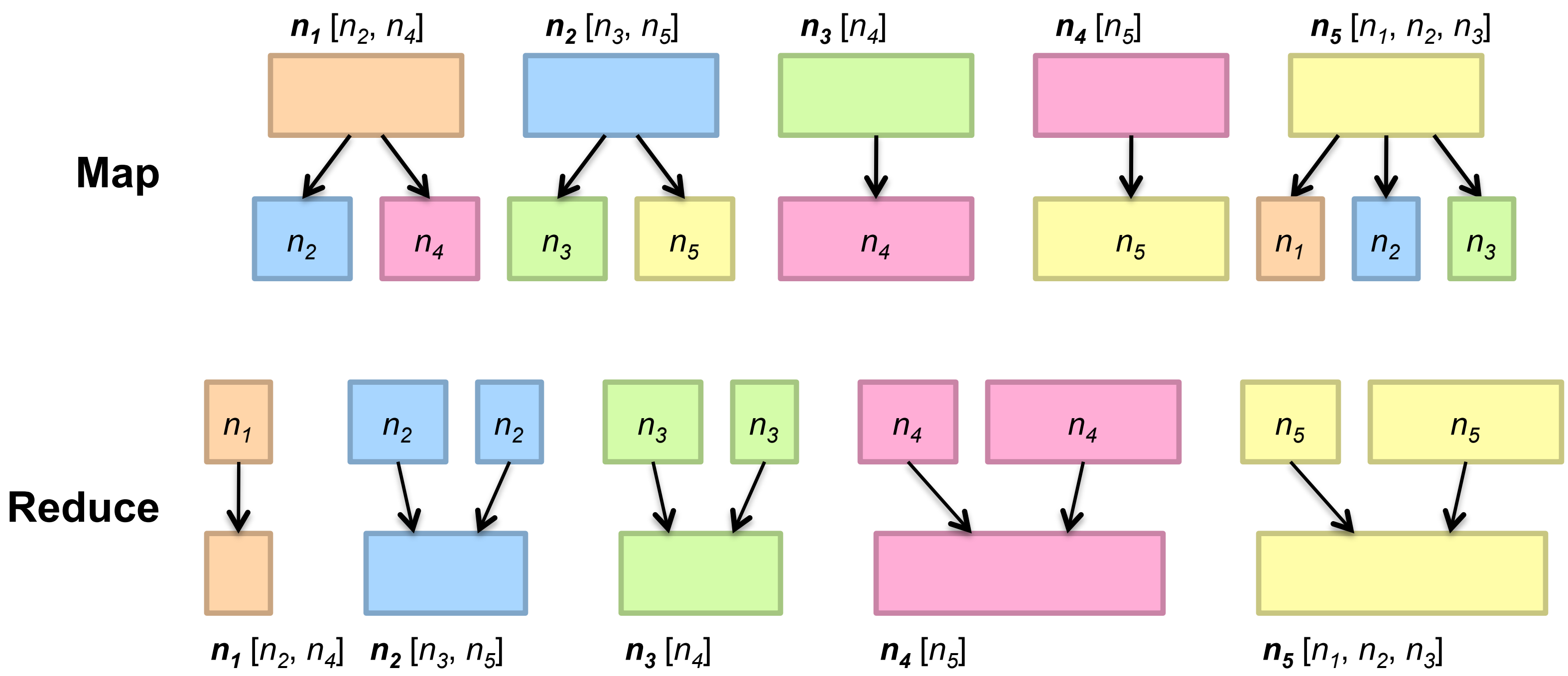


Sample PageRank Iteration (2)





PageRank in MapReduce



PageRank Pseudo-Code

```
class Mapper
    method Map(nid n, node N)
        p ← N.PageRank/|N.AdjacencyList|
        Emit(nid n, N)                              ▷ Pass along graph structure
        for all nodeid m ∈ N.AdjacencyList do
            Emit(nid m, p)                          ▷ Pass PageRank mass to neighbors

class Reducer
    method Reduce(nid m, [p1, p2, ...])
        M ← ∅
        for all p ∈ counts [p1, p2, ...] do
            if IsNode(p) then
                M ← p                               ▷ Recover graph structure
            else
                s ← s + p                           ▷ Sums incoming PageRank contributions
        M.PageRank ← s
        Emit(nid m, node M)
```

PageRank vs. BFS

	PageRank	BFS
Map	PR/N	d+1
Reduce	sum	min

Complete PageRank

- Two additional complexities
  - What is the proper treatment of dangling nodes?
  - How do we factor in the random jump factor?
- Solution:
  - Second pass to redistribute "missing PageRank mass" and account for random jumps

$$p' = \alpha \left(\frac{1}{N} \right) + (1 - \alpha) \left(\frac{m}{N} + p \right)$$

  - *p* is PageRank value from before, *p*' is updated PageRank value
  - *N* is the number of nodes in the graph
  - *m* is the missing PageRank mass
- Additional optimization: make it a single pass!

PageRank Convergence

- Alternative convergence criteria
  - Iterate until PageRank values don't change
  - Iterate until PageRank rankings don't change
  - Fixed number of iterations
- Convergence for web graphs?
  - Not a straightforward question
- Watch out for link spam:
  - Link farms
  - Spider traps
  - …

Beyond PageRank

- Variations of PageRank
  - Weighted edges
  - Personalized PageRank
- Variants on graph random walks
  - Hubs and authorities (HITS)
  - SALSA

Applications

- Static prior for web ranking
- Identification of “special nodes” in a network
- Link recommendation
- Additional feature in any machine learning problem

Other Classes of Graph Algorithms

- Subgraph pattern matching
- Computing simple graph statistics
  - Degree vertex distributions
- Computing more complex graph statistics
  - Clustering coefficients
  - Counting triangles

General Issues for Graph Algorithms

- Sparse vs. dense graphs
- Graph topologies

Source: http://www.flickr.com/photos/fusedforces/4324320625/

MapReduce Sucks

- Java verbosity
- Hadoop task startup time
- Stragglers
- Needless graph shuffling
- Checkpointing at each iteration

Iterative Algorithms

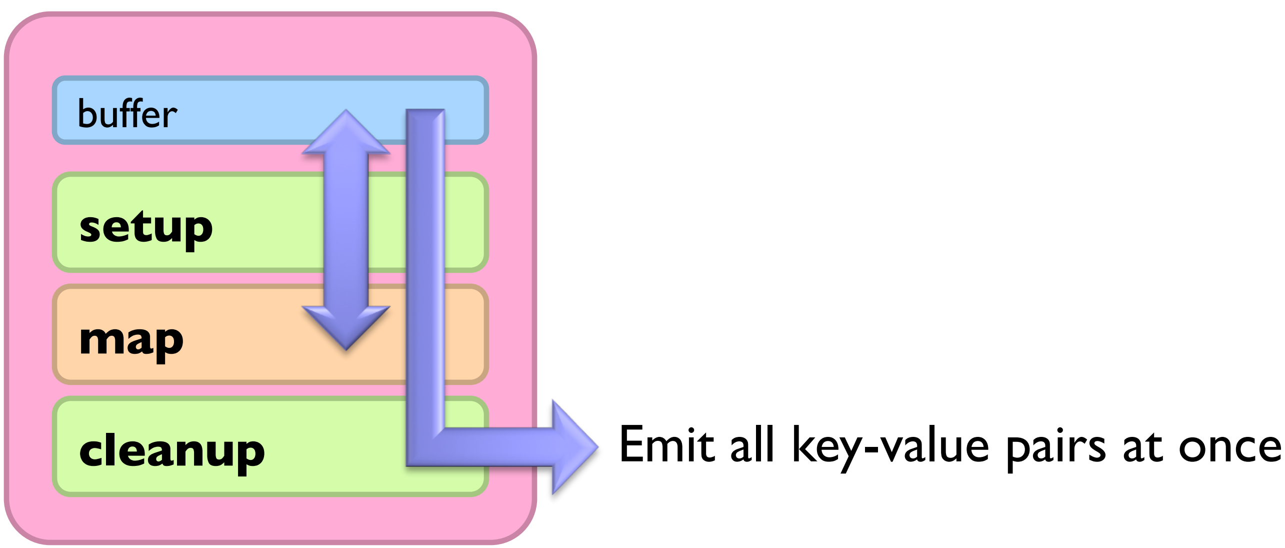
Source: Wikipedia (Water wheel)

MapReduce sucks at iterative algorithms

- Alternative programming models (later)
- Easy fixes (now)

In-Mapper Combining

- Use combiners
  - Perform local aggregation on map output
  - Downside: intermediate data is still materialized
- Better: in-mapper combining
  - Preserve state across multiple map calls, aggregate messages in buffer, emit buffer contents at end
  - Downside: requires memory management

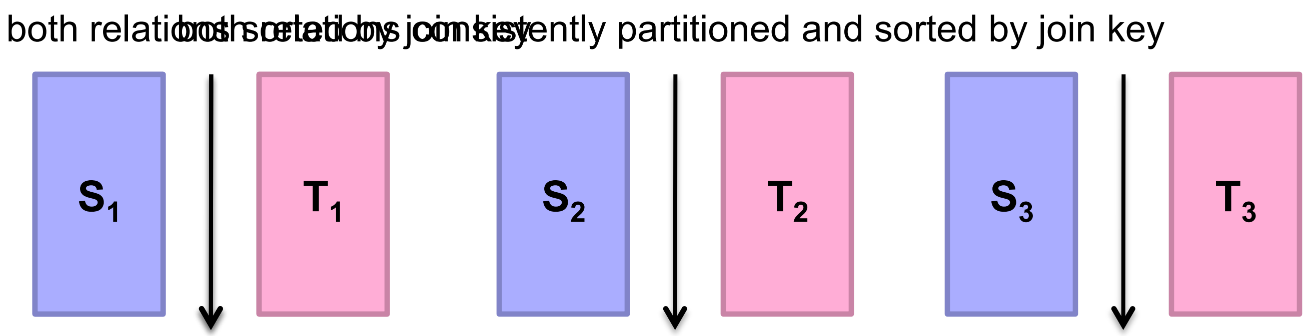


Better Partitioning

- Default: hash partitioning
  - Randomly assign nodes to partitions
- Observation: many graphs exhibit local structure
  - E.g., communities in social networks
  - Better partitioning creates more opportunities for local aggregation
- Unfortunately, partitioning is **hard**!
  - Sometimes, chick-and-egg…
  - But cheap heuristics sometimes available
  - For webgraphs: range partition on domain-sorted URLs

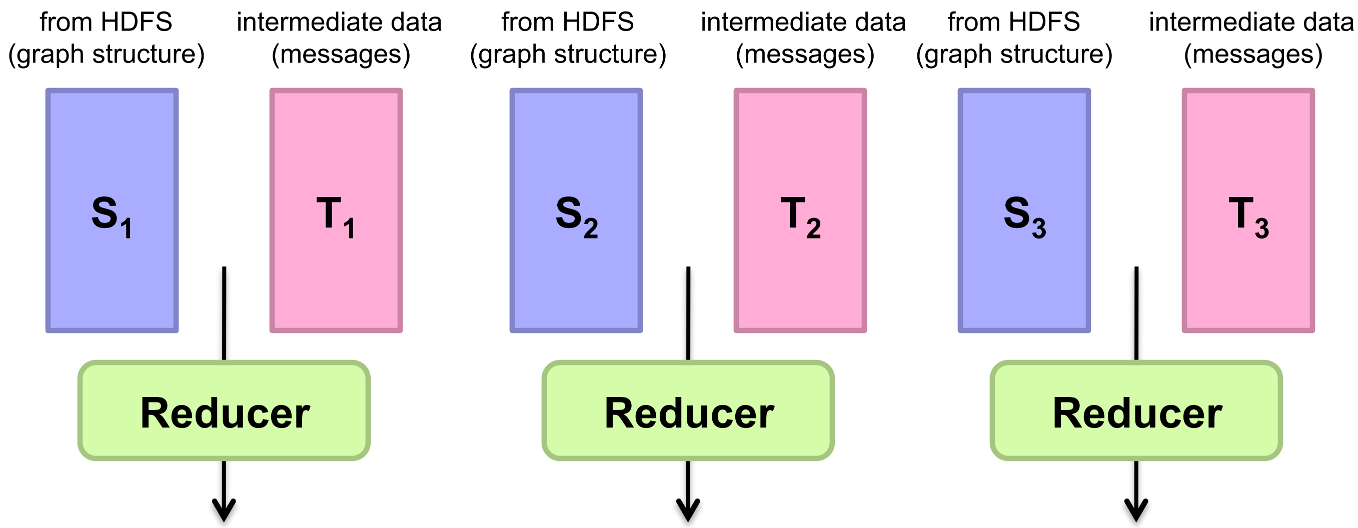
Schimmy Design Pattern

- Basic implementation contains two dataflows:
  - Messages (actual computations)
  - Graph structure (“bookkeeping”)
- Schimmy: separate the two dataflows, shuffle only the messages
  - Basic idea: merge join between graph structure and messages



Do the Schimmy!

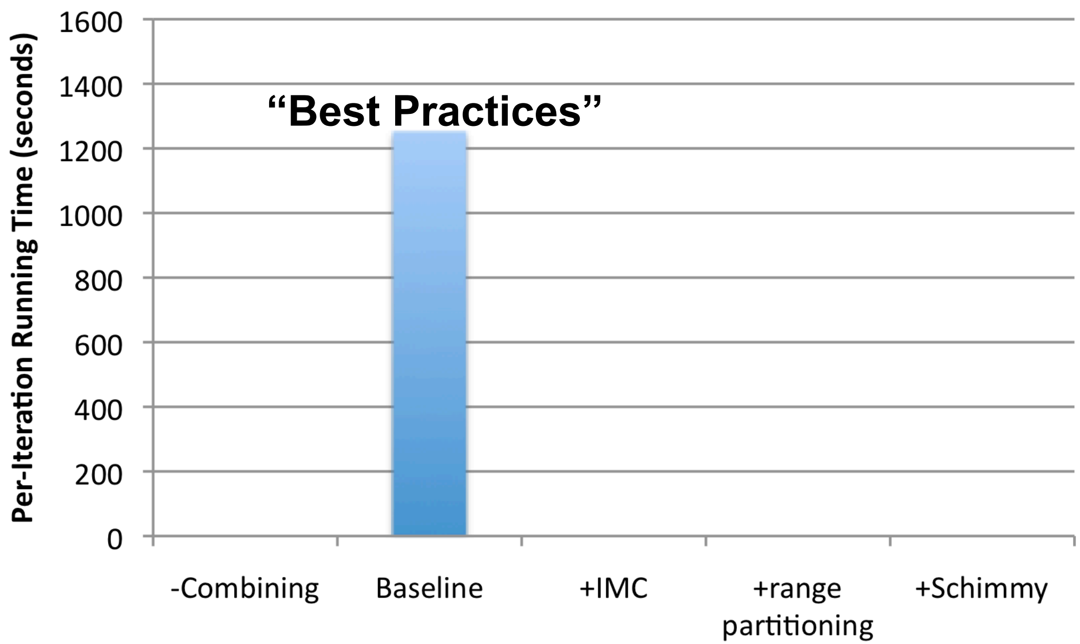
- Schimmy = reduce side parallel merge join between graph structure and messages
  - Consistent partitioning between input and intermediate data
  - Mappers emit only messages (actual computation)
  - Reducers read graph structure directly from HDFS



Experiments

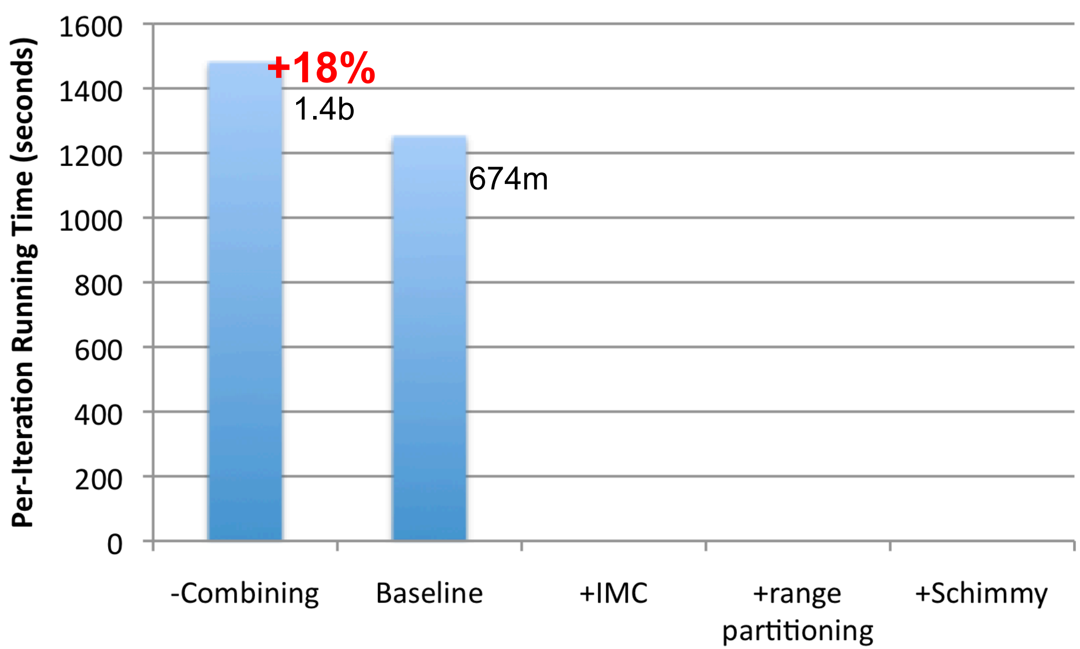
- Cluster setup:
  - 10 workers, each 2 cores (3.2 GHz Xeon), 4GB RAM, 367 GB disk
  - Hadoop 0.20.0 on RHELS 5.3
- Dataset:
  - First English segment of ClueWeb09 collection
  - 50.2m web pages (1.53 TB uncompressed, 247 GB compressed)
  - Extracted webgraph: 1.4 billion links, 7.0 GB
  - Dataset arranged in crawl order
- Setup:
  - Measured per-iteration running time (5 iterations)
  - 100 partitions

Results



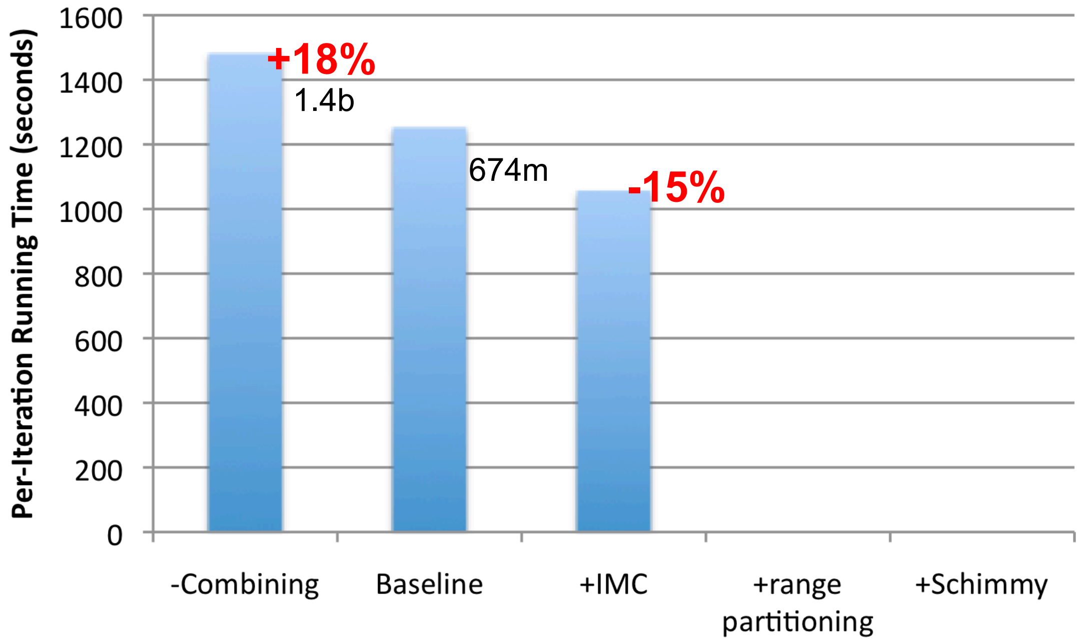
Per-Iteration Running Time (seconds)
1600
1400
1200
1000
800
600
400
200
0
“Best Practices”
-Combining
Baseline
+IMC
+range partitioning
+Schimmy

Results



Per-Iteration Running Time (seconds)
1600
1400
1200
1000
800
600
400
200
0
+18%
1.4b
674m
-Combining
Baseline
+IMC
+range partitioning
+Schimmy

Results



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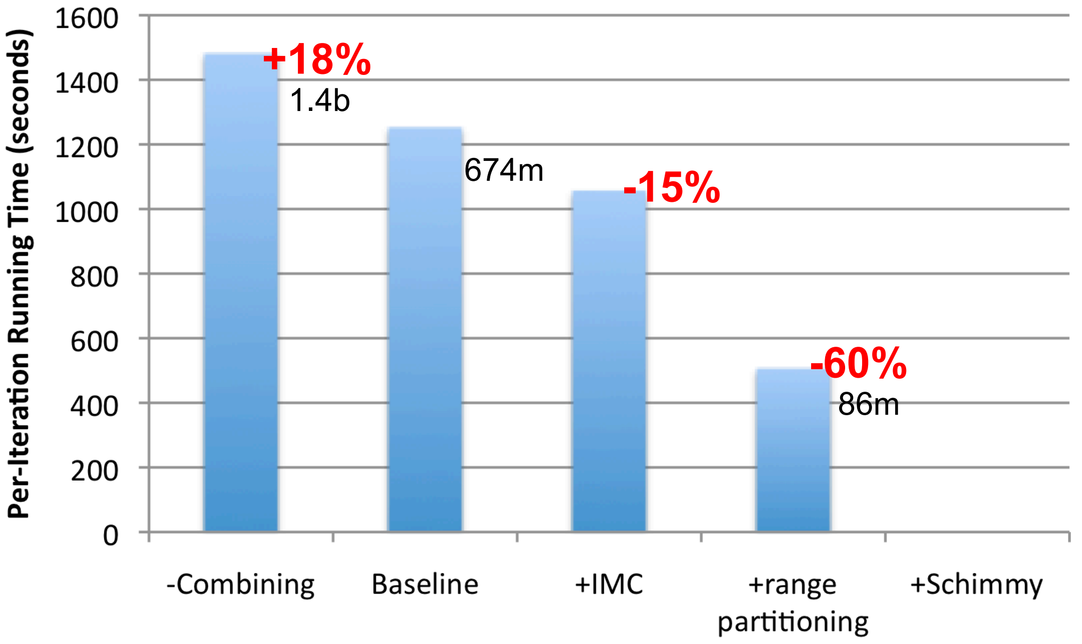

Per-Iteration Running Time (seconds)
1600
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1.4b
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-15%
-60%
86m
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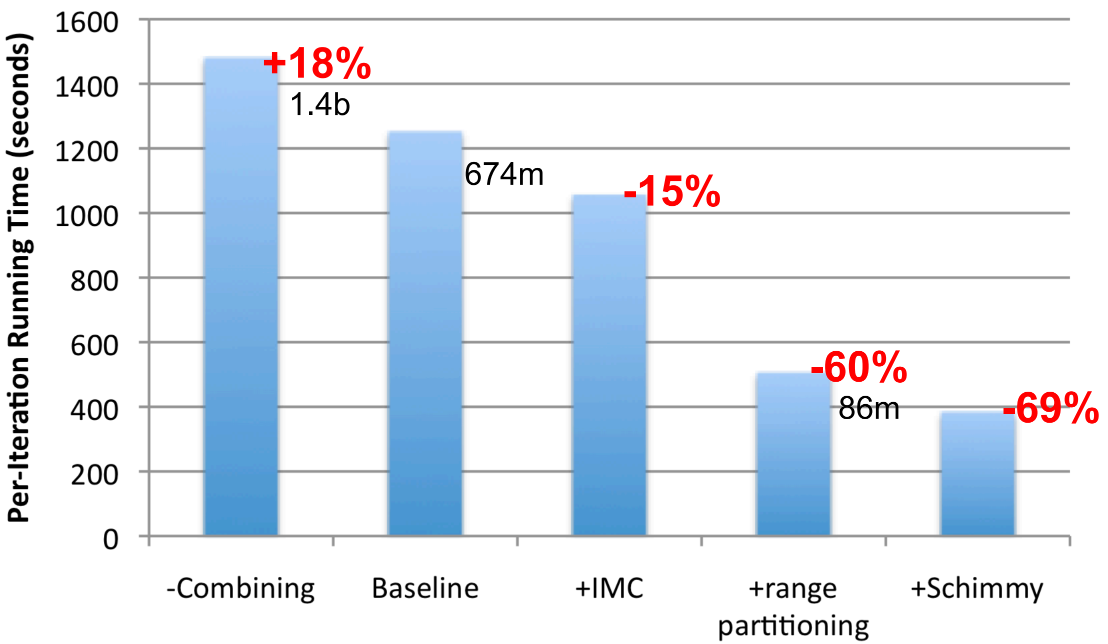
Results



Per-Iteration Running Time (seconds)
1600
1400
1200
1000
800
600
400
200
0
+18%
1.4b
674m
-15%
-60%
86m
-69%
-Combining
Baseline
+IMC
+range partitioning
+Schimmy

MapReduce sucks at iterative algorithms

- Alternative programming models (later)
- Easy fixes (now)

Later, the "hammer" argument…

Today's Agenda

- Graph problems and representations
- Parallel breadth-first search
- PageRank
- Beyond PageRank and other graph algorithms
- Optimizing graph algorithms

Source: Wikipedia (Japanese rock garden)