Cloud Computing Lecture #5

Graph Algorithms with MapReduce

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Wednesday, October 1, 2008

Today's Topics

- Introduction to graph algorithms and graph representations
- Single Source Shortest Path (SSSP) problem
  - Refresher: Dijkstra's algorithm
  - Breadth-First Search with MapReduce
- PageRank

What's a graph?

- G = (V,E), where
  - V represents the set of vertices (nodes)
  - E represents the set of edges (links)
  - Both vertices and edges may contain additional information
- Different types of graphs:
  - Directed vs. undirected edges
  - Presence or absence of cycles
- Graphs are everywhere:
  - Hyperlink structure of the Web
  - Physical structure of computers on the Internet
  - Interstate highway system
  - Social networks

Some Graph Problems

- Finding shortest paths
  - Routing Internet traffic and UPS trucks
- Finding minimum spanning trees
  - Telco laying down fiber
- Finding Max Flow
  - Airline scheduling
- Identify "special" nodes and communities
  - Breaking up terrorist cells, spread of avian flu
- Bipartite matching
  - Monster.com, Match.com
- And of course... PageRank

Graphs and MapReduce

- Graph algorithms typically involve:
  - Performing computation at each node
  - Processing node-specific data, edge-specific data, and link structure
  - Traversing the graph in some manner
- Key questions:
  - How do you represent graph data in MapReduce?
  - How do you traverse a graph in MapReduce?

Representing Graphs

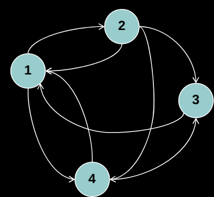
- G = (V, E)
  - A poor representation for computational purposes
- Two common representations
  - Adjacency matrix
  - Adjacency list

Adjacency Matrices

Represent a graph as an *n* x *n* square matrix *M*

- $n = |V|$
- $M_{ij} = 1$ means a link from node *i* to *j*

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0



Adjacency Matrices: Critique

- Advantages:
  - Naturally encapsulates iteration over nodes
  - Rows and columns correspond to inlinks and outlinks
- Disadvantages:
  - Lots of zeros for sparse matrices
  - Lots of wasted space

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Adjacency Lists

Take adjacency matrices… and throw away all the zeros

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

→

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

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Adjacency Lists: Critique

- Advantages:
  - Much more compact representation
  - Easy to compute over outlinks
  - Graph structure can be broken up and distributed
- Disadvantages:
  - Much more difficult to compute over inlinks

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Single Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
- First, a refresher: Dijkstra's Algorithm

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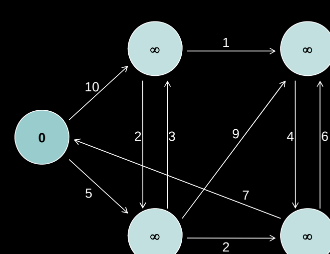
Dijkstra's Algorithm Example



Example from CLR

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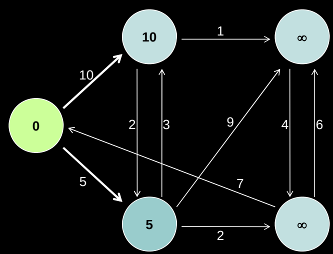
Dijkstra's Algorithm Example



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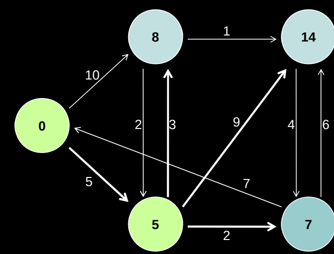
Dijkstra's Algorithm Example



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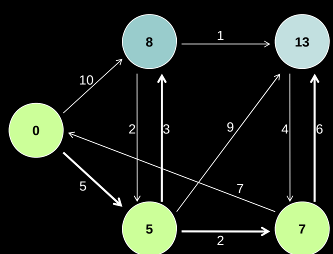
Dijkstra's Algorithm Example



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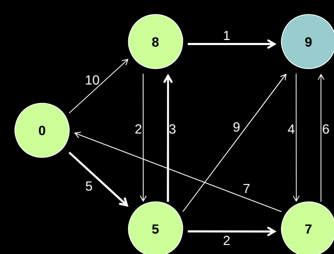
Dijkstra's Algorithm Example



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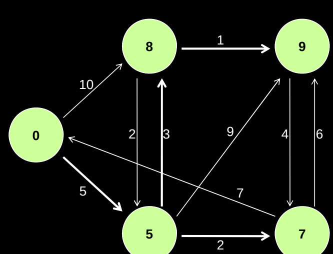
Dijkstra's Algorithm Example



Example from CLR

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Single Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
- Single processor machine: Dijkstra's Algorithm
- MapReduce: parallel Breadth-First Search (BFS)

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Finding the Shortest Path

- First, consider equal edge weights
- Solution to the problem can be defined inductively
- Here's the intuition:
  - DistanceTo(startNode) = 0
  - For all nodes *n* directly reachable from startNode, DistanceTo(*n*) = 1
  - For all nodes *n* reachable from some other set of nodes S, DistanceTo(n) = 1 + min(DistanceTo(m), m $\in$ S)

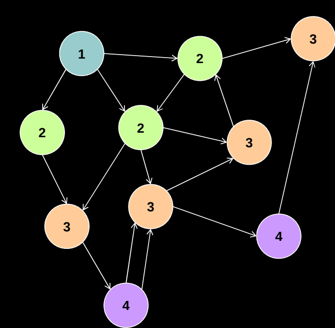
From Intuition to Algorithm

- A map task receives
  - Key: node *n*
  - Value: D (distance from start), points-to (list of nodes reachable from *n*)
- $\forall p \in$ points-to: emit (*p*, D+1)
- The reduce task gathers possible distances to a given *p* and selects the minimum one

Multiple Iterations Needed

- This MapReduce task advances the "known frontier" by one hop
  - Subsequent iterations include more reachable nodes as frontier advances
  - Multiple iterations are needed to explore entire graph
  - Feed output back into the same MapReduce task
- Preserving graph structure:
  - Problem: Where did the points-to list go?
  - Solution: Mapper emits (*n*, points-to) as well

Visualizing Parallel BFS



Termination

- Does the algorithm ever terminate?
  - Eventually, all nodes will be discovered, all edges will be considered (in a connected graph)
- When do we stop?

Weighted Edges

- Now add positive weights to the edges
- Simple change: points-to list in map task includes a weight *w* for each pointed-to node
  - emit (*p*, D+w_p) instead of (*p*, D+1) for each node *p*
- Does this ever terminate?
  - Yes! Eventually, no better distances will be found. When distance is the same, we stop
  - Mapper should emit (*n*, D) to ensure that "current distance" is carried into the reducer

Comparison to Dijkstra

- Dijkstra's algorithm is more efficient
  - At any step it only pursues edges from the minimum-cost path inside the frontier
- MapReduce explores all paths in parallel
  - Divide and conquer
  - Throw more hardware at the problem

General Approach

- MapReduce is adapt at manipulating graphs
  - Store graphs as adjacency lists
- Graph algorithms with for MapReduce:
  - Each map task receives a node and its outlinks
  - Map task compute some function of the link structure, emits value with target as the key
  - Reduce task collects keys (target nodes) and aggregates
- Iterate multiple MapReduce cycles until some termination condition
  - Remember to "pass" graph structure from one iteration to next

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Random Walks Over the Web

- Model:
  - User starts at a random Web page
  - User randomly clicks on links, surfing from page to page
- What's the amount of time that will be spent on any given page?
- This is PageRank

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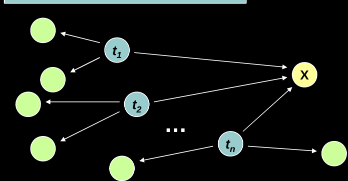
PageRank: Defined

Given page x with in-bound links $t_1 \ldots t_n$, where

- $C(t)$ is the out-degree of t
- α is probability of random jump
- N is the total number of nodes in the graph

$$PR(x) = \alpha\left(\frac{1}{N}\right) + (1-\alpha)\sum_{i=1}^{n}\frac{PR(t_i)}{C(t_i)}$$



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Computing PageRank

- Properties of PageRank
  - Can be computed iteratively
  - Effects at each iteration is local
- Sketch of algorithm:
  - Start with seed PR_i values
  - Each page distributes PR_i "credit" to all pages it links to
  - Each target page adds up "credit" from multiple in-bound links to compute PR_{i+1}
  - Iterate until values converge

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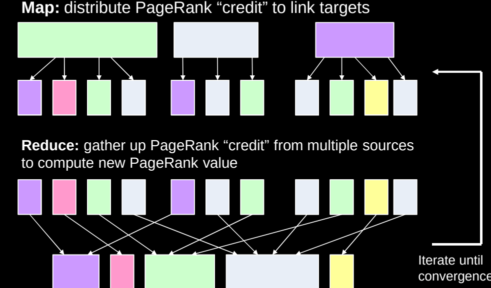
PageRank in MapReduce

Map: distribute PageRank "credit" to link targets



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PageRank: Issues

- Is PageRank guaranteed to converge? How quickly?
- What is the “correct” value of α, and how sensitive is the algorithm to it?
- What about dangling links?
- How do you know when to stop?

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Questions?

(Ask them now, because you’re going to have to implement this!)